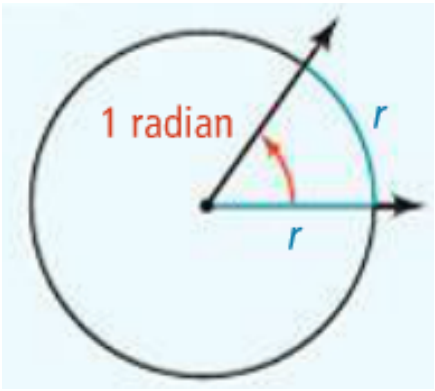
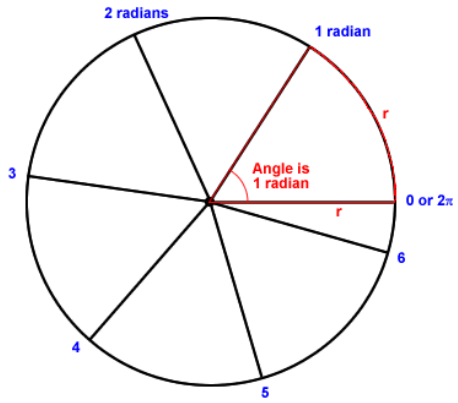
13.3 - Radian Measure

A **radian** is the measure of a central angle that intercepts an arc with length equal to the radius of the circle. Radians, like degrees, measure the amount of rotation from the initial side to the terminal side of an angle.





Apr 6-8:38 AM

take note

Key Concept Proportion Relating Radians and Degrees

You can use the proportion $\frac{d^\circ}{180^\circ} = \frac{r \text{ radians}}{\pi \text{ radians}}$ to convert between radians and degrees.

Here's Why It Works

Because the circumference of a circle is $2\pi r$, there are 2π radians in any circle. Since 2π radians $= 360^\circ$, it follows that π radians $= 180^\circ$. This equality leads to the following *conversion factors* for converting between radian measure and degree measure.

take note

Key Concept Converting Between Radians and Degrees

To convert degrees to radians, multiply by $\frac{\pi \text{ radians}}{180^\circ}$.

To convert radians to degrees, multiply by $\frac{180^\circ}{\pi \text{ radians}}$.

Apr 6-8:54 AM

Ex1. What is the **radian** measure of the following angles?

$60^\circ \cdot \frac{\pi}{180} \rightarrow \boxed{\frac{\pi}{3}}$

$-135^\circ \cdot \frac{\pi}{180} \rightarrow \boxed{-\frac{3\pi}{4}}$

$\frac{60}{180}$ simplify

$\frac{1}{3}$

Apr 6-8:55 AM

Ex2. What is the **degree** measure of the following angles?

$\frac{5\pi}{6} \cdot \frac{180}{\pi}$

$\frac{900}{6}$

$\boxed{150^\circ}$

$\frac{-7\pi}{4} \cdot \frac{180}{\pi}$

$\frac{-1260}{4}$

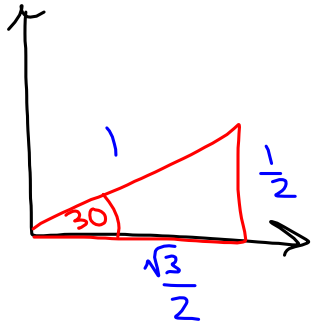
$\boxed{-315^\circ}$

Apr 6-9:03 AM

Finding Exact Values of Trig. Ratios

Ex.3 What is the exact sine, cosine and tangent of the following angles?

$\frac{\pi}{6} \cdot \frac{180}{\pi} \rightarrow 30°$



sin: $\frac{1}{2}$ cos: $\frac{\sqrt{3}}{2}$

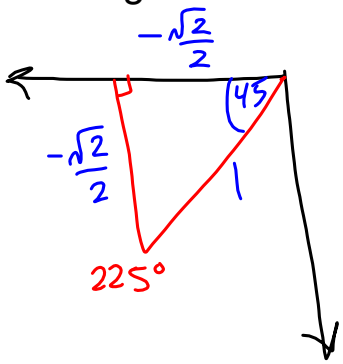
tan: $\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \rightarrow \frac{\sqrt{3}}{3}$

Apr 6-9:11 AM

Finding Exact Values of Trig. Ratios

Ex.4 What is the exact sine, cosine and tangent of the following angles?

$\frac{5\pi}{4} \cdot \frac{180}{\pi} \rightarrow 225°$



sin: $-\frac{\sqrt{2}}{2}$ cos: $-\frac{\sqrt{2}}{2}$

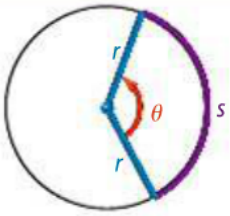
tan: 1

Apr 6-9:11 AM

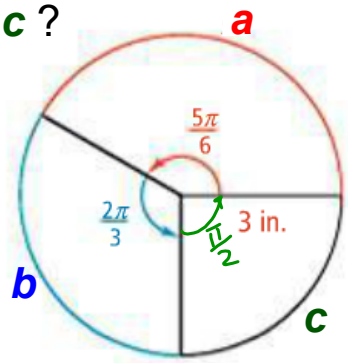
take note

Key Concept Length of an Intercepted Arc

For a circle of radius r and a central angle of measure θ (in radians), the length s of the intercepted arc is $s = r\theta$.



Ex.5 What is the length of **a** and **b** *and* **c** ?



$a = (3)\left(\frac{5\pi}{6}\right) \approx 7.85$ in.

$b = (3)\left(\frac{2\pi}{3}\right) \approx 6.28$ in.

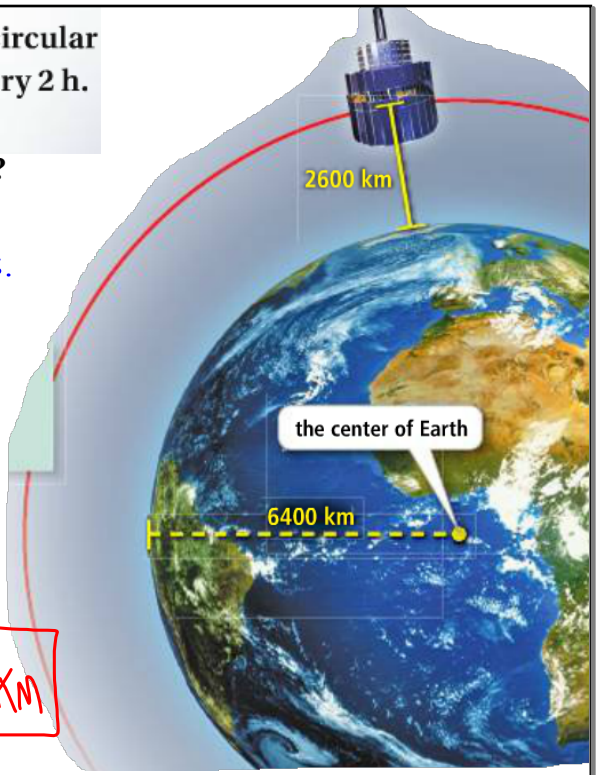
$c = (3)\left(\frac{\pi}{2}\right) \approx 4.71$ in.

Apr 6-9:15 AM

Weather Satellite A weather satellite in a circular orbit around Earth completes one orbit every 2 h.

a. How far does the satellite travel in 1 h?

b. How far does the satellite travel in 20 minutes?



a.) 1 hour → 180° → π rads.

radius = 9000 km

$S = (9000)(\pi)$

$S \approx 28,274.33$ km

b.) $S = (9000)\left(\frac{\pi}{3}\right) \approx 9,424.78$ km

Apr 6-9:17 AM

HW: p. 848:

#'s 6 - 35 all, 37 - 44 all, 47

Apr 6-9:21 AM