7-2: Properties of Exponential Functions

***Objectives:** 1. To graph exponential functions with transformations.*
2. Calculate half-life and continuous interest problems.

$$y = a(b)^{x-h} + k$$

Multiplier:
Stretch $a > 1$
Shrink $0 < a < 1$
Reflect if negative

Vertical Shift:
down/up

Horizontal Shift:
left/right
(remember opp sign!)

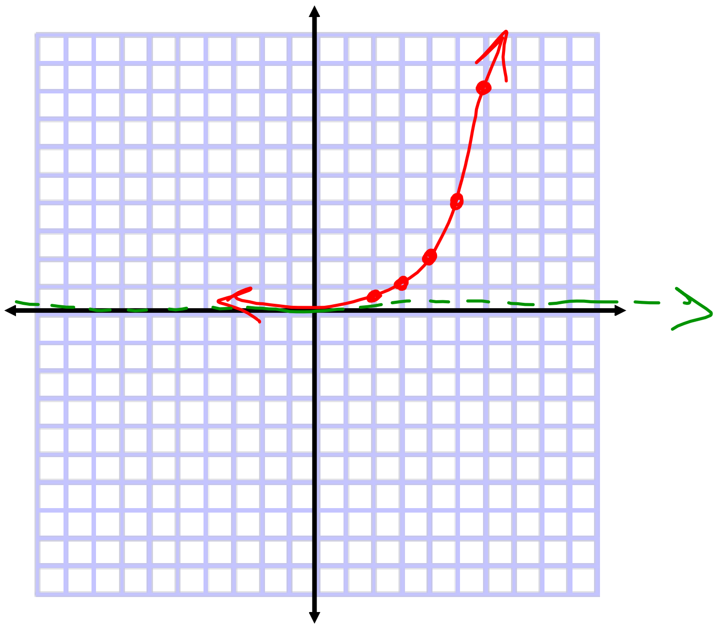
Feb 14-9:49 AM

Graphing Exponential Functions

Ex. $y = (2)^{x-3}$

1. Graph $y = (2)^x$ as your parent function first.

H.A.: $y=0$



2. Shift the graph according to *h* and *k* values.

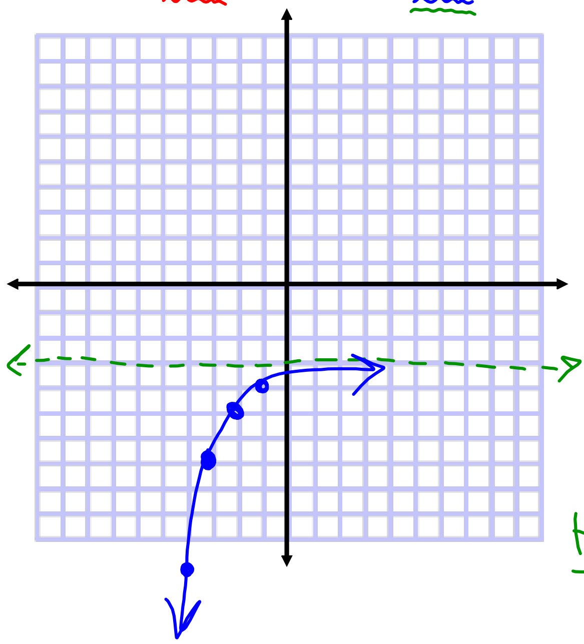
Right 3

D: ℝ R: $y>0$

Feb 14-9:59 AM

Graphing Exponential Functions

Ex. $y = -2(1/2)^{x+2} - 3$



1. Graph $y = (1/2)^x$ as your parent function first.

2. Apply a-value.
Reflect over x-axis, strech by 2.

3. Shift the graph according to h and k values.
Left 2, down 3

HA: $y=-3$

D: $\mathbb{R}$

R: $y<-3$

Feb 14-10:01 AM

Half-life Model:

$$y = a(0.5)^{t/k}$$

number of time periods (t)

initial value/amount

half-life constant:

k = time for a half-life

Ex.
Isotope H-3 has a half-life of 8 days. Write a function for a 1000-mg sample. Then use the function to find out how much remains after 50 days.

$$y = 1000(0.5)^{\left(\frac{50}{8}\right)} \approx \boxed{13.14 \text{ mg}}$$

Feb 14-10:08 AM

Ex.
Phosphorus -13 has a half-life of 20 days. Write a function for a 72-mg sample. Then use the function to find out how much remains after 31 days.

$y = 72(0.5)^{(31/20)}$

$\boxed{y \approx 24.59 \text{ mg}}$

Mar 6-9:33 AM

The Number "e":

A naturally occurring constant, like *pi*. Its used as a base of many real-life problems involving certain growth and decay.

Lets try it out in our calculators... *(e^x)-button usually*

$e^1 = 2.718....$

$e^2 = 7.389...$

$e^{(3/2)} = 4.482...$

Feb 15-9:21 AM

Continuous Model:

Instead of calculating interest on a finite number of periods, such as yearly or monthly, continuous compounding calculates interest assuming constant compounding over an infinite number of periods. Also used in certain growth and decay situations.

Amount (after t years)

$$y = P \cdot e^{rt}$$

time (in years)

Principal (initial value/amount)

Interest Rate (convert to decimal)

Ex.
Suppose you invest $1050 at an annual interest rate of 6.5% compounded continuously. How much will you have after 7 years?

$y = 1050 \cdot e^{\wedge}(0.065 \cdot 7)$

$\boxed{y \approx \$1,654.98}$

Feb 14-10:16 AM

Ex.
Suppose you invest $120,000 at an ~~annual~~ interest rate of 3% compounded continuously.

a. How much will you have after 18 months? → 1.5 = t yrs.

$y = 120000 \cdot e^{\wedge}(.03 \cdot 1.5)$

$\boxed{y = \$125,523.34}$

b. How many years until you have over half a million dollars?

$500,000 > ?

$\boxed{47 \rightarrow 48 \text{ yrs}}$

Feb 14-11:30 AM

HW:

p. 447:

#'s: 11, 16, 20, 21, 23 - 30, 33, 36, 41

Feb 14-11:39 AM