

• KEY.

Name: _____ Per: _____

Solving Polynomial Functions with Various Factoring Methods

Directions: In your groups of 3 – 4 people, go through this packet working on one round at a time. Make sure everyone in your group understands the problems as you go! Use academic vocabulary throughout the activity and take time to have collaborative discussions.

Academic Vocabulary terms:

These are terms that you should be using when having discussions with each other.

Factor	Zeros	Multiplicity	Zero Product Property	Solutions	GCF (Greatest Common Factor)
Difference of Two Squares			Sum/Difference of Cubes	Quadratic Formula	Factor by Grouping
Real/imaginary Solutions			Synthetic Division	Remainder	The Remainder Theorem
The Rational Root Theorem			The Conjugate Root Theorem	Degree	Coefficients
					Complex Roots

<p>Quadratic Formula:</p> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	<p>Sum/Difference of Cubes:</p> <p>Sum: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$</p> <p>Diff: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$</p>
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Round 1:

$x^2 + 7x = 30$
 $x^2 + 7x - 30 = 0$
 $(x + 10)(x - 3) = 0$
 $x = -10$ $x = 3$

$18x^2 + 12x = 0$ **GCF!**
 $6x(3x + 2) = 0$
 $6x = 0$ $3x + 2 = 0$
 $x = 0$ $3x = -2$
 $x = -2/3$

$4x^2 - 25 = 0$
 $(2x + 5)(2x - 5) = 0$
 $x = -5/2$ $x = 5/2$

Diff. of Two Squares!

$x^2 - 5 = -3x$
 $x^2 + 3x - 5 = 0$ **Quadratic Formula!**
 $x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-5)}}{2(1)}$
 $x = \frac{-3 \pm \sqrt{29}}{2}$

Round 2:

$$x^4 - 125x = 0$$

$$x(x^3 - 125) = 0$$

$$x(x-5)(x^2+5x+25) = 0$$

Diff. of
2 cubes!

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(25)}}{2(1)}$$

$$x = 0$$

$$x = 5$$

$$x = \frac{-5 \pm \sqrt{-75}}{2}$$

$$x = \frac{-5 \pm 5i\sqrt{3}}{2}$$

$$27x^3 + 8 = 0$$

$$a = 3x \quad b = 2$$

sum of
cubes!

$$(3x+2)(9x^2-6x+4) = 0$$

$$3x+2=0$$

$$3x = -2$$

$$x = -\frac{2}{3}$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(9)(4)}}{2(9)}$$

$$x = \frac{6 \pm \sqrt{-108}}{18}$$

$$x = \frac{6 \pm 6i\sqrt{3}}{18}$$

$$x = \frac{1 \pm i\sqrt{3}}{3}$$

Round 3:

$$x^4 - 2x^2 = 63$$

$$x^4 - 2x^2 - 63 = 0$$

$$(x^2 - 9)(x^2 + 7) = 0$$

$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

$$x^2 + 7 = 0$$

$$x^2 = -7$$

$$x = \pm i\sqrt{7}$$

$$5x^7 + 50x^5 + 80x^3 = 0$$

$$5x^3(x^4 + 10x^2 + 16) = 0$$

$$5x^3(x^2 + 8)(x^2 + 2) = 0$$

$$x^2 + 8 = 0$$

$$x^2 = -8$$

$$x = \pm 2i\sqrt{2}$$

$$x^2 + 2 = 0$$

$$x^2 = -2$$

$$x = \pm i\sqrt{2}$$

$$x = 0 \text{ (mult. 3)}$$

$$(x^3 + 6x - 17) \div (x - 2)$$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 6 & -17 \\ & & 2 & 4 & 20 \\ \hline & 1 & 2 & 10 & 3 \end{array}$$

$$x^2 + 2x + 10, R 3$$

$$\begin{array}{r} 4x^4 - 22x^3 + 3x^2 + 38x - 6 \\ \hline x - 5 \end{array}$$

$$\begin{array}{r|rrrrr} 5 & 4 & -22 & 3 & 38 & -6 \\ & & 20 & -10 & -25 & 15 \\ \hline & 4 & -2 & -7 & 3 & 9 \end{array}$$

NO

$$\begin{array}{r} x^3 - 4x^2 - 11x + 30 \\ \hline x + 3 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & -4 & -11 & 30 \\ & & -3 & 21 & -20 \\ \hline & 1 & -7 & 10 & 10 \end{array}$$

$$x^2 - 7x + 10$$

$$(x - 2)(x - 5)(x + 3)$$

Round 5:

$$x = 0 \text{ (mult. 2)}, x = 3, x = -\frac{1}{2}$$

$$\begin{array}{l} x^2 = 0 \quad x - 3 = 0 \quad 2x = -1 \\ \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 2x + 1 = 0 \end{array}$$

$$x^2(x - 3)(2x + 1) = 0 \quad \begin{array}{l} -6x \\ 1x \end{array}$$

$$x^2(2x^2 - 5x - 3) = 0$$

$$2x^2 - 5x^3 - 3x^2 = 0$$

$$x = \pm 3i$$

$$x = \pm \sqrt{5}$$

$$x^2 = 9i^2$$

$$x^2 = 5$$

$$x^2 = -9$$

$$x^2 - 5 = 0$$

$$x^2 + 9 = 0$$

$$(x^2 + 9)(x^2 - 5) = 0$$

$$x^4 + 4x^2 - 45 = 0 \quad \begin{array}{l} 9x^2 \\ -5x^2 \end{array}$$

Round 6:

Given zeros of $x = -1$ and $x = 2$

Solve: $x^4 - 4x^3 + 6x^2 + x - 10 = 0$

$$\begin{array}{r|rrrrr} -1 & 1 & -4 & 6 & 1 & -10 \\ & & -1 & 5 & -11 & 10 \\ \hline & 1 & -5 & 11 & -10 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -5 & 11 & -10 \\ & & 2 & -6 & 10 \\ \hline & 1 & -3 & 5 & 0 \end{array}$$

$$x^2 - 3x + 5 = 0$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{-11}}{2}$$

$$x = \frac{3 \pm i\sqrt{11}}{2}$$

$$x = 2$$

$$x = -1$$

Round 7:

Given zeros of $x = -2$ and $x = 1$ (mult. 2)

Solve: $x^5 + x^3 + 2x^2 - 12x + 8 = 0$

$$\begin{array}{r|rrrrrr} -2 & 1 & 0 & 1 & 2 & -12 & 8 \\ & & -2 & 4 & -10 & 16 & -8 \\ \hline & 1 & -2 & 5 & -8 & 4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & 5 & -8 & 4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & 4 & -4 & 0 \end{array}$$

$$\begin{array}{r|rrr} 1 & 1 & 0 & 4 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$$x^2 + 4 = 0$$

$$x^2 = -4$$

$$x = \pm 2i$$

$$x = 1 \text{ (mult. 2)}$$

$$x = -2$$

Challenge Problem #1:

$$x^4 + 2x^3 - 27x - 54 = 0 \quad \text{Use Factor}$$

by grouping!

$$x^3(x+2) - 27(x+2) = 0$$

$$(x^3 - 27)(x+2) = 0$$

Diff. of
Two cubes!

$$(x-3)(x^2+3x+9)(x+2) = 0$$

$$x = 3$$

$$x = -2$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{-27}}{2}$$

$$x = \frac{-3 \pm 3i\sqrt{3}}{2}$$

$$x = 3$$

$$x = -2$$

Challenge Problem #2:

$$x^6 + 19x^3 - 216 = 0 \quad \text{Quad.-like}$$

Factoring!

-8	27
19	-216
Sum	

$$(x^3 - 8)(x^3 + 27) = 0$$

Sum AND Difference
of cubes!

$$(x-2)(x^2+2x+4)(x+3)(x^2-3x+9) = 0$$

$$x = 2$$

$$x = -3$$

$$x = \frac{-2 \pm \sqrt{-12}}{2}$$

$$x = \frac{-2 \pm 2i\sqrt{3}}{2}$$

$$x = \frac{3 \pm \sqrt{-27}}{2}$$

$$x = -1 \pm i\sqrt{3}$$

$$x = \frac{3 \pm 3i\sqrt{3}}{2}$$

Challenge Problem #3:

$$x = 3 \pm 5i$$

$$x - 3 = \pm 5i$$

$$x^2 - 6x + 9 = 25i^2$$

$$x^2 - 6x + 9 = -25$$

$$x^2 - 6x + 34 = 0$$

$$x = i\sqrt{2}$$

$$x^2 = i^2 \cdot 2$$

$$x^2 = -2$$

$$x^2 + 2 = 0$$

$$x = \pm 3i$$

$$x^2 = 9i^2$$

$$x^2 = -9$$

$$x^2 + 9 = 0$$

$$(x^2 + 2)(x^2 + 9) = 0$$

$$(x^4 + 11x^2 + 18)(x^2 - 6x + 34)$$

$$x^6 - 6x^5 + 34x^4$$

$$+ 11x^4 - 66x^3 + 374x^2$$

$$+ 18x^2$$

$$- 108x$$

$$+ 612$$

↓ ✓ ✓ ✓

$$0 = x^6 - 6x^5 + 45x^4 - 66x^3 + 392x^2 - 108x + 612$$

Challenge Problem #4:

Given $x = -2$ (mult. 3) as a zero, solve...

$$x^5 + 6x^4 + 17x^3 + 38x^2 + 60x + 40$$

$$\begin{array}{r|rrrrrr} -2 & 1 & 6 & 17 & 38 & 60 & 40 \end{array}$$

$$\begin{array}{r} -2 & -8 & -18 & -40 & -20 \end{array}$$

$$\begin{array}{r|rrrrr} 1 & 4 & 9 & 20 & 20 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 4 & 9 & 20 \end{array}$$

$$\begin{array}{r} -2 & -4 & -10 & -20 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 2 & 5 & 10 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 5 & 10 \end{array}$$

$$\begin{array}{r} -2 & 0 & -6 \end{array}$$

$$\begin{array}{r|rrr} 1 & 0 & 5 & 0 \end{array}$$

Solve:

$$x^2 + 5 = 0$$

$$x^2 = -5$$

$$x = \pm i\sqrt{5}$$

$$x = -2 \text{ (mult. 3)}$$

