• KEY •

UNIT REVIEW 4 – Systems of Equations and Inequalities

Name: ______________________

Part I: 4.1 – Systems of Linear Equations KEY

1.) Solve $\begin{cases} y = 4x + 1 \\ 3x + 2y = 13 \end{cases}$

$3x + 2(4x+1) = 13$

$3x + 8x + 2 = 13$

$11x = 11$

$x = 1$

$y = 4(1) + 1$

$y = 5$

$\boxed{(1, 5)}$

2.) Solve $\begin{cases} 4x - 8y = 16 \\ -6y + 3x = 12 \end{cases}$

rearrange...

$3(4x - 8y = 16)$ $\quad 3\,4x - 8y = 16$

$-4(3x - 6y = 12)$

$12x - 24y = 48$

$-12x + 24y = -48$

$0 = 0$

$\boxed{\text{Infinite Solutions}}$

3.) Solve $\begin{cases} +4(5x + 3y = 1) \\ -3(3x + 4y = -6) \end{cases}$

$20x + 12y = 4$

$-9x - 12y = 18$

$11x = 22$

$x = 2$

$5(2) + 3y = 1$

$10 + 3y = 1$

$3y = -9$

$y = -3$

$\boxed{(2, -3)}$

4.) Solve $\begin{cases} 2y = 7 + 6x \\ 3x - y = 9 \end{cases}$

$-6x + 2y = 7$

$2(3x - y = 9)$

$6x - 2y = 18$

$0 = 25$

$\boxed{\text{No Solution}}$

5.) With the wind, you can fly 2160 miles in 3 hours. Against the same wind, you can only fly 1800 miles, but in 4 hours. Find your plane's average velocity in still wind and the average velocity of the wind.

$x + y = 720 = \frac{2160}{3}$

$x - y = 450 \quad \frac{1800}{4}$

$2x = 1170$

$x = 585$

Answer: avg. velocity in still wind: $\boxed{585 \text{ mph}}$, wind avg. velocity: $\boxed{135 \text{ mph}}$

6.) Kim and Pranav are selling pies for a school fundraiser. Customers can buy blueberry pies and blackberry pies. Kim sold 4 blueberry pies and 5 blackberry pies for a total of \$117.80. Pranav sold 5 blueberry pies and 14 blackberry pies for a total of \$263.50. Find the cost each of one blueberry pie and one blackberry pie.

$-5(4x + 5y = 117.80) \rightarrow -20x - 25y = -589$

$4(5x + 14y = 263.50) \rightarrow 20x + 56y = 1054$

$31y = 465$

$y = 15$

$4x + 5(15) = 117.80$

$4x = 42.8$

Answer: Price of 1 **blueberry** pie: $\boxed{\$10.70}$, Price of 1 **blackberry** pie: $\boxed{\$15.00}$

7.) A company is planning to manufacture cheap affordable computer desks ... will be $60,000 and it will cost $200 to produce each desk. Then each desk will be sold for $450.

a.) Write a cost function, ***C***, of producing ***x*** desks: $\boxed{= 200x + 60000}$

b.) Write a revenue function, ***R***, from the sale of ***x*** desks: $\boxed{= 450x}$

c.) Determine the break-even point. Write the point as an ordered pair, then **you must explain** (thoroughly) the meaning of the break-even point in the context of this situation.

Show Work Here for part c. :

$450x = 200x + 60000$
$250x = 60000$
$x = 240 \rightarrow y = 450(240)$
$y = 108000$

If the company can make and sell 240 desks, it will cost \$108000, but the company will take in \$108000 from the sales. Thus, breaking-even.

Break-Even Point *(as an ordered pair)*: $\boxed{(240, 108000)}$

8.) A chemist needs to mix an alloy with a 34% silver nitrate content and an alloy with a 4% silver nitrate content to obtain 100 ounces of a new alloy of 7% silver nitrate content. How many ounces of each of the original alloys (the 34% one and the 4% one) must be used to achieve this?

$$\begin{cases} x + y = 100 \\ .34x + .04y = 7 \end{cases} \leftarrow (100)(.07)$$

$-.04x - .04y = -4$
$+ .34x + .04y = 7$

$.30x = 3$
$x = 10$

Answer: Amount of the **34% alloy**: $\boxed{10 \text{ ounces}}$, Amount of the **4% alloy**: $\boxed{90 \text{ ounces}}$

9.) Solve this 3-variable system.

$-12(2x - y + z = 1) \rightarrow -24x + 12y - 12z = -12$
$3(3x - 3y + 4z = 5) \rightarrow 9x - 9y + 12z = 15$
$4(4x - 2y + 3z = 4) \rightarrow 16x - 8y + 12z = 16$

①+②: $4(-15x + 3y) = 3 \rightarrow -60x + 12y = 12$
①+③: $-3(-8x + 4y) = 4 \rightarrow 24x - 12y = -12$

$-36x = 0$
$x = 0$

$-8(0) + 4y = 4$
$4y = 4$
$y = 1$

$2(0) - 1 + z = 1$
$z = 2$

$\boxed{(0, 1, 2)}$

10.) Solve this 3-variable system.

$-2(x + 2y - z = 5) \rightarrow -2x - 4y + 2z = -10$
$2x - y + 3z = 0 \rightarrow 2x - y + 3z = 0$
$-2y + z = 1$

$-5y + 5z = -10$
$-5(-2y + z = 1)$

~~$-2y + (-3) = 1$~~
~~$-2y = 4$~~
~~$y = -2$~~

$-5y + 5z = -10$
$10y - 5z = -5$

$5y = -15$
$y = -3$

~~$x + 2(-2) - (-3) = 5$~~
~~$x - 4 + 3 = 5$~~
~~$x = 6$~~

$\boxed{(6, -3, -5)}$

$-2(-3) + z = 1$
$6 + z = 1$
$z = -5$

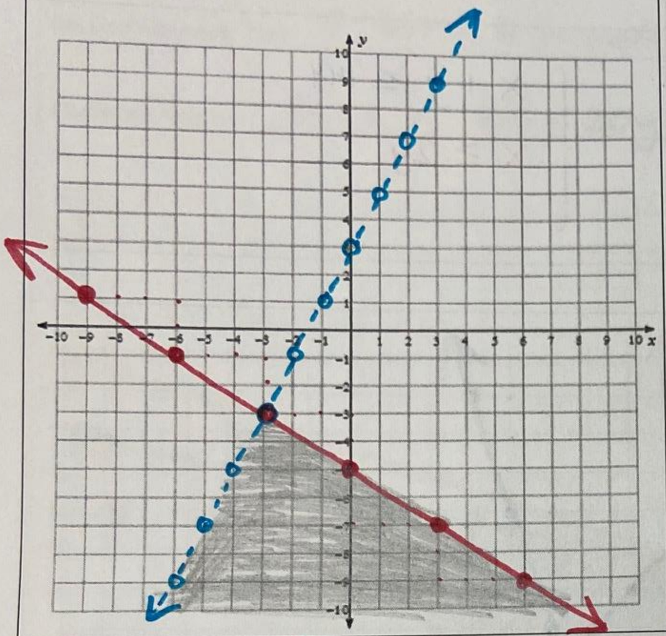
$x + 2(-3) + 5 = 5$
$x = 6$

day,
15

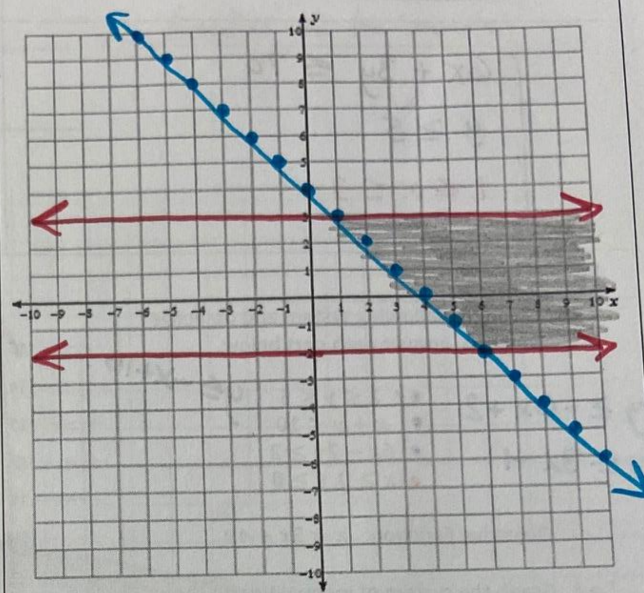
Part II: 4.2 – Systems of Inequalities and Linear Programming KEY

Graph each system of inequalities. Be sure to have accurate line types (*solid or dashed*) as well as shading.

1.) $\begin{cases} y \le -\frac{2}{3}x - 5 \\ y < 2x + 3 \end{cases}$



2.) $\begin{cases} -2 \le y \le 3 \\ y \ge -x + 4 \end{cases}$



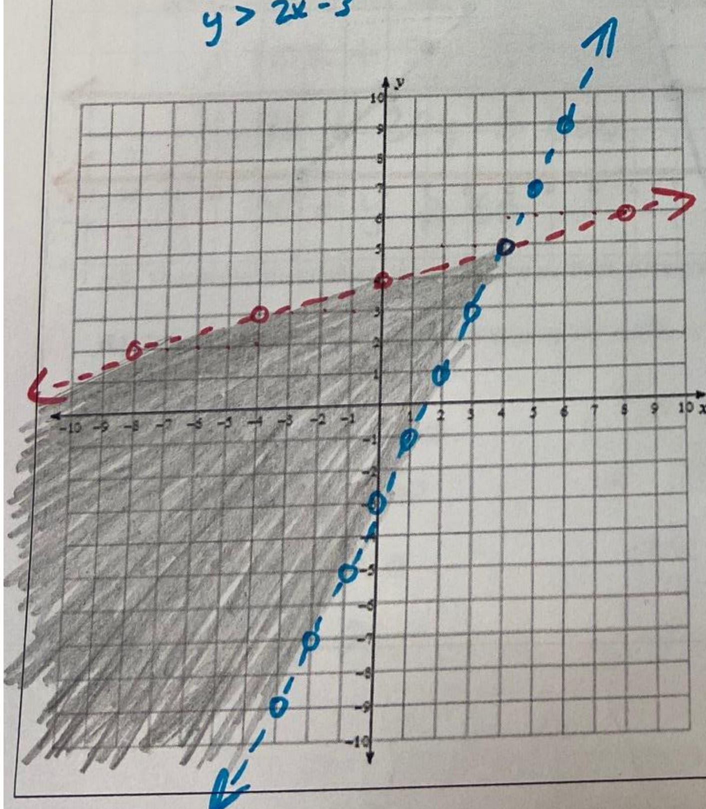
3.) $\begin{cases} x - 4y > -16 \\ 2x - y < 3 \end{cases}$

$-4y > -x - 16$

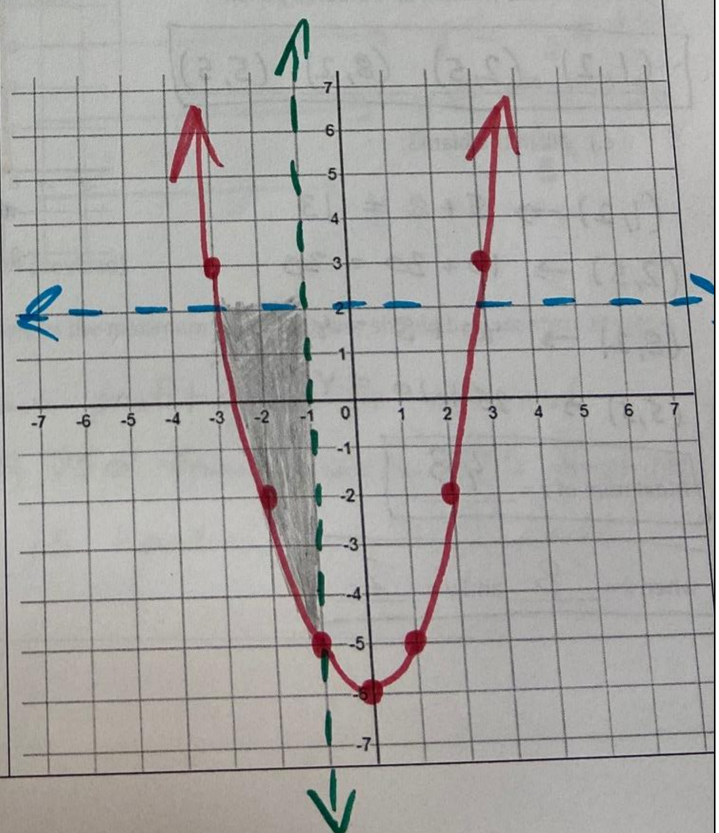
$y < \frac{1}{4}x + 4$

$-y < -2x + 3$

$y > 2x - 3$



4.) $\begin{cases} y \ge x^2 - 6 \\ y < 2 \\ x < -1 \end{cases}$



Write the correct system of inequalities, using x and y, that correctly models each situation.

5.) You have a bunch of dirty dress pants and shirts. You have at most \$70 to spend at the dry cleaners today, where pants cost \$6 and shirts \$3 to get cleaned. You need at least 5 shirts for the week. Also, at least 1 and at most 3 pairs of pants for the week as well.	6.) For your baseball card collection, you have at most room for 144 cards, split between two brands; Fleer and Tops. You want to have at least 2 times as many Fleer cards as Tops cards.
$\begin{cases} 6x + 3y \leq 70 \\ y \geq 5 \\ 1 \leq x \leq 3 \end{cases}$	$\begin{cases} x + y \leq 144 \\ x \geq 2y \end{cases}$

7.) Given the following system and objective function, answer each part below.

$-2y \geq -6x + 2$

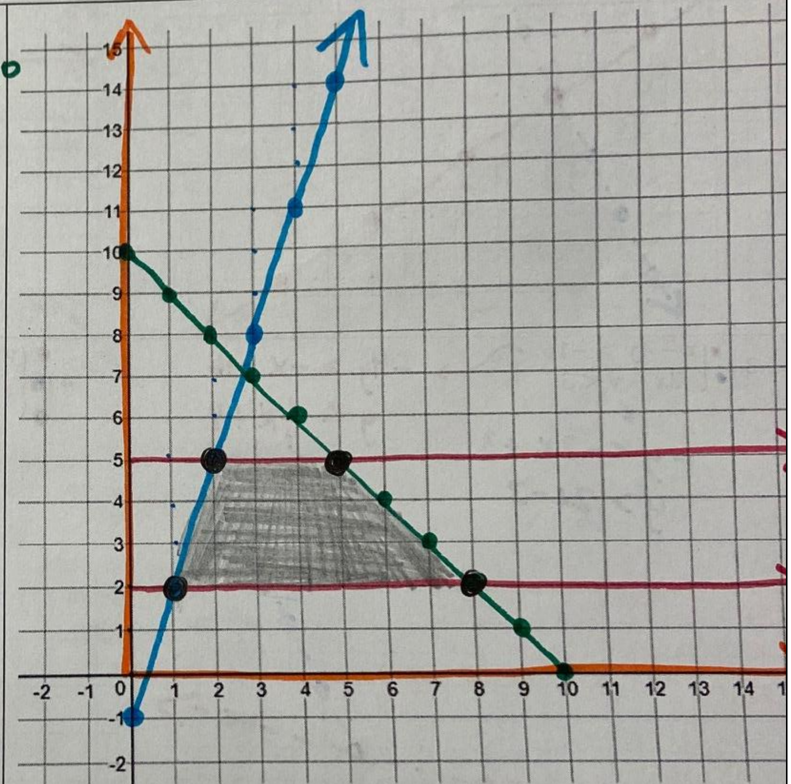
$y \leq 3x - 1$

$$\begin{cases} 2 \leq y \leq 5 \\ x + y \leq 10 \\ 6x - 2y \geq 2 \\ x \geq 0, y \geq 0 \end{cases}$$

$y \leq -x + 10$

Objective Function: $z = 5x + 4y$

a.) Graph the system of inequalities on the graph to the right. →



b.) Find and list the coordinates of each vertex of the feasible region:

List of vertices (*written as 4 ordered pairs*):

$\boxed{(1,2)\quad(2,5)\quad(8,2)\quad(5,5)}$

c.) Fill in the blanks:

$(1,2) \rightarrow 5 + 8 = 13$

$(2,5) \rightarrow 10 + 20 = 30$

$(8,2) \rightarrow 40 + 8 = 48$

$(5,5) \rightarrow 25 + 20 = 45$

Maximum of z = 48

when x = 8 and y = 2

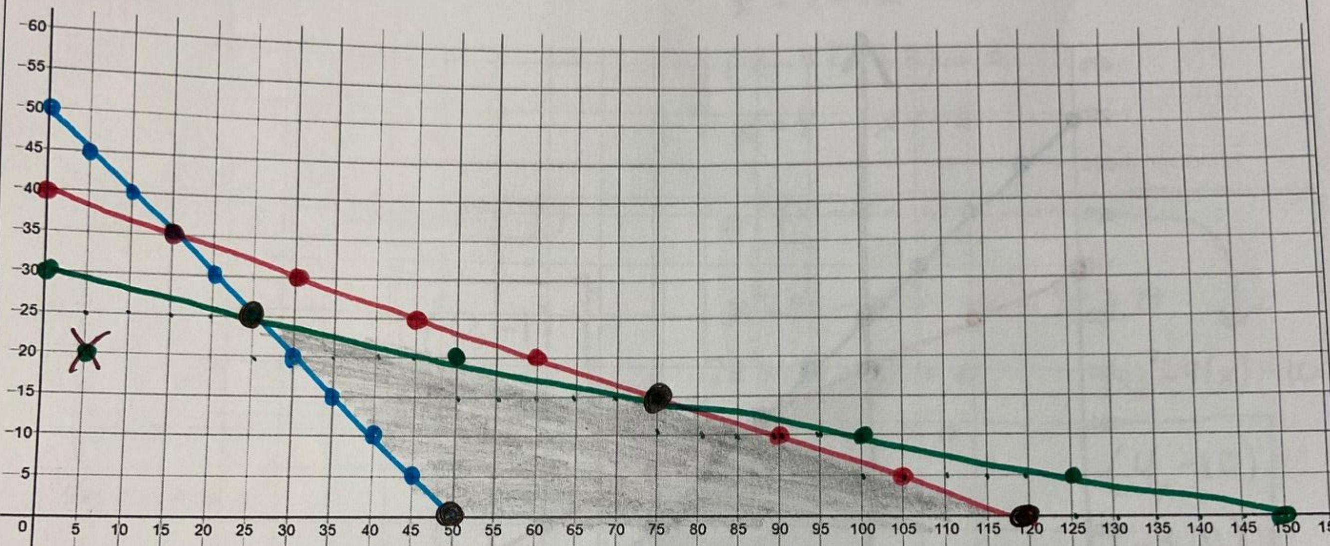
20 hours → 1200 min.

8.) You are screen printing t-shirts and hoodies to sell. You have at most 20 hours to make t-shirts and hoodies. You also have no more than $600 to spend. Finally, you want to have at least 50 items to sell. T-shirts take 10 minutes each to make, cost you $4, and yield a profit of $6 per t-shirt. Hoodies take 30 minutes each to make, cost you $20, and yield a profit of $20 per hoodie. How many of each should be made and sold to maximize profit? What is the maximum profit?

a.)

Set your variables: x = t-shirts to be made   y = hoodies to be made

Objective Function: Profit = 6x + 20y



b.) Write a system of linear inequalities that models this situation.

$$\begin{cases} 10x + 30y \le 1200 & \longrightarrow y \le -\frac{1}{3}x + 40 \\ 4x + 20y \le 600 & \longrightarrow y \le -\frac{1}{5}x + 30 \\ x + y \ge 50 & \longrightarrow y \ge -x + 50 \end{cases}$$

c.) Graph the inequalities, then label the vertices. (Use graph provided)

d.) What is the x and y values that maximize the profit, as well as the maximum profit? Answer should be in context as well as a complete sentence.

(50,0) → $300

(25,25) → $650

(120,0) → $720

(75,15) → $750

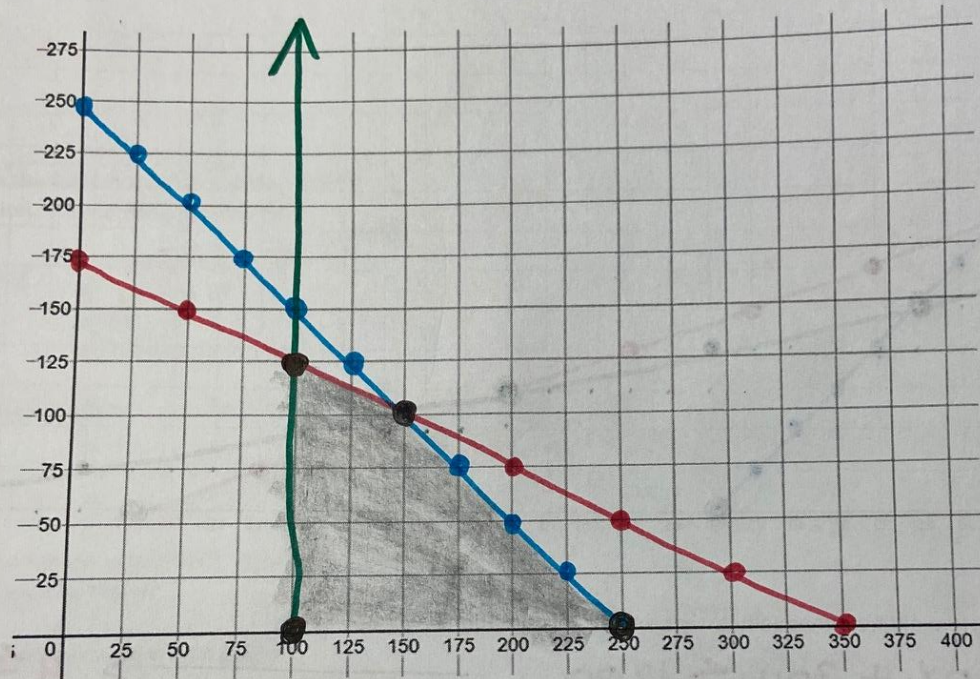
Maximum profit can be obtained of $750 from making 75 t-shirts and 15 hoodies.

9.) A merchant plans to sell two models of home computers at costs $200 and $400 respectively. The $200 model yields a profit of $45 and the $400 model yields a profit of $50. The merchant estimates that the total monthly demand for both computers together will not exceed 250 units. Finally, the merchant wants to have stocked at least 100 units of the $200 model. Find the number of units of each model that should be stocked in order to maximize profit. Assume that the merchant will not invest more than $70,000 in costs to the computer inventory.

a.)

Set your variables: x = $200 model computer y = $400 model computer

Objective Function: Profit = $45x + 50y$



a.) Write a system of linear inequalities that models this situation.

$$\begin{cases} 200x + 400y \leq 70000 \\ x + y \leq 250 \\ x \geq 100 \end{cases}$$

b.) Graph the inequalities, then label the vertices. (Use graph provided)

c.) What is the x and y values that maximize the profit, as well as the maximum profit? Answer should be in context as well as a complete sentence.

$(100, 0) \rightarrow 4500$
$(250, 0) \rightarrow 11250$
$(100, 125) \rightarrow 10750$
$(150, 100) \rightarrow 11750$

Maximum profit of $11,750 can be obtained from stocking 150 of the $200 computer model and 100 of the $400 computer model.

Part III: 4.3 – Nonlinear Systems of Equations KEY

1.) $\begin{cases} 3x - y = 9 \\ x^2 - 2y = 10 \end{cases}$ $\rightarrow y = 3x-9$

$x^2 - 2(3x-9) = 10$

$x^2 - 6x + 18 = 10$

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$x = 4 \quad x = 2$

$\boxed{(4,3)\ (2,-3)}$

2.) $\begin{cases} x - y = 1 \\ x^2 + y^2 = 25 \end{cases}$ $\rightarrow y = x-1$

$x^2 + (x-1)^2 = 25$

$x^2 + x^2 - 2x + 1 = 25$

$2x^2 - 2x - 24 = 0$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4 \quad x = -3$

$\boxed{(4,3)\ (-3,-4)}$

3.) $\begin{cases} x^2 - 4y = 4 \\ x + y = -1 \end{cases}$ $\rightarrow y = -x-1$

$x^2 - 4(-x-1) = 4$

$x^2 + 4x + 4 = 4$

$x^2 + 4x = 0$

$x(x+4) = 0$

$x = 0 \quad x = -4$

$\boxed{(0,-1)\ (-4,3)}$

4.) $\begin{cases} x^2 - 4x - 10 = y \\ -x^2 - 2x + 14 = y \end{cases}$

$x^2 - 4x - 10 = -x^2 - 2x + 14$

$2x^2 - 2x - 24 = 0$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4 \quad x = -3$

$(x)^2 - 4(x) - 10 =$

$\boxed{(4,-10)\ (-3,11)}$

5.) $4(2x^2 + 3y^2 = 21)$
$3(3x^2 - 4y^2 = 23)$

$8x^2 + 12y^2 = 84$

$+\ 9x^2 - 12y^2 = 69$

$17x^2 = 153$

$x^2 = 9$

$x = \pm 3$

$2(3)^2 + 3y^2 = 21$

$2(9) + 3y^2 = 21$

$18 + 3y^2 = 21$

$3y^2 = 3$

$y^2 = 1$

$y = \pm 1$

$\boxed{(3,1)\ (3,-1)\ (-3,1)\ (-3,-1)}$

6.) $\begin{cases} (x-1)^2 + (y+1)^2 = 5 \\ 2x - y = 3 \end{cases}$ $\rightarrow y = 2x-3$

$(x-1)^2 + (2x-2)^2 = 5$

$x^2 - 2x + 1 + 4x^2 - 8x + 4 = 5$

$5x^2 - 10x = 0$

$5x(x-2) = 0$

$x = 0, \ x = 2$

$\boxed{(0,-3)\ (2,1)}$

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7.) $\begin{cases} 3x^2 + 2y^2 = 14 \\ 2(2x^2 - y^2 = 7) \end{cases}$

$$3x^2 + 2y^2 = 14$$
$$+\ 4x^2 - 2y^2 = 14$$
$$7x^2 = 28$$
$$x^2 = 4$$
$$x = \pm 2$$

$$3(2)^2 + 2y^2 = 14$$
$$3(4) + 2y^2 = 14$$
$$12 + 2y^2 = 14$$
$$2y^2 = 2$$
$$y^2 = 1$$
$$y = \pm 1$$

$\boxed{(2,1)\ (2,-1)\ (-2,1)\ (-2,-1)}$

8.) $\begin{cases} x - 2y = 4 \\ 2y^2 + xy = 8 \end{cases}$ $\rightarrow$ $x = 2y + 4$

$$2y^2 + y(2y+4) = 8$$
$$2y^2 + 2y^2 + 4y = 8$$
$$4y^2 + 4y - 8 = 0$$
$$y^2 + y - 2 = 0$$
$$(y+2)(y-1) = 0$$
$$y = -2,\ y = 1$$

$\boxed{(0,-2)\ (6,1)}$

9.) $\begin{cases} xy = 1 \\ -2x + y = 1 \end{cases}$ $\rightarrow$ $y = 2x + 1$

$$x(2x+1) = 1$$
$$2x^2 + x - 1 = 0$$
$$(2x-1)(x+1) = 0$$
$$x = \tfrac{1}{2} \qquad x = -1$$

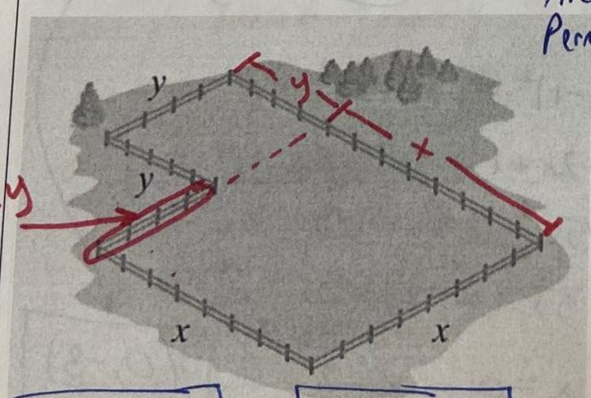
$\boxed{(\tfrac{1}{2}, 2)\ (-1,-1)}$

10.) $\begin{cases} y^2 = 4x \\ x - 2y + 3 = 0 \end{cases}$ $\rightarrow$ $x = 2y - 3$

$$y^2 = 4(2y-3)$$
$$y^2 = 8y - 12$$
$$y^2 - 8y + 12 = 0$$
$$(y-6)(y-2) = 0$$
$$y = 6 \quad y = 2$$

$\boxed{(9,6)\ (1,2)}$

11.) Two adjoining square fields with an area of 2900 square feet are to be enclosed with 240 feet of fencing. The situation is represented in the figure. Find the length of each side where a variable appears. *(There may be more than one answer).*



Area: $x^2 + y^2 = 2900$
Perm: $4x + 2y = 240$

$$2y = -4x + 240$$
$$y = -2x + 120$$

$$x^2 + (-2x+120)^2 = 2900$$
$$x^2 + 4x^2 - 480x + 14400 = 2900$$
$$5x^2 - 480x + 11500 = 0$$
$$x^2 - 96x + 2300 = 0$$
$$(x-50)(x-46) = 0$$
$$x = 50 \quad x = 46$$

$\boxed{x = 50 \text{ ft.},\ y = 20 \text{ ft.}}$ or $\boxed{x = 46 \text{ ft.},\ y = 28 \text{ ft.}}$