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Name: _____

Date: _____

Per: _____

UNIT 5 Review - Part 5.1: Graphing Polynomial Functions and Real Zeros

Given the following polynomial functions, get them into standard form, then find the following unknowns.

<p>1.) $y = 13 - 2x + 4x^5 + 2x - 2$</p> <p>Standard Form: $4x^5 + 11$</p> <p>End Behavior: <u>Down</u> / <u>Up</u></p>	<p>2.) $y = 2x^2(-3x^2 + 2x) - 3(x - 7) - 4x^3$</p> <p>$-6x^4 + 4x^3 - 3x + 21 - 4x^3$</p> <p>Standard Form: $-6x^4 - 3x + 21$</p> <p>End Behavior: <u>Down</u> / <u>Down</u></p>
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Factor each polynomial function and then find all the zeros. Make sure to state multiplicity of multiple zeros.

<p>3.) $f(x) = x^3 + 2x^2 - 49x - 98$</p> <p>$x^2(x+2) - 49(x+2) = 0$</p> <p>$(x^2 - 49)(x+2) = 0$</p> <p>$(x+7)(x-7)(x+2) = 0$</p> <p>$x = -7, 7, -2$</p>	<p>4.) $g(x) = -4x^4 - 12x^3 + 40x^2$</p> <p>$-4x^2(x^2 + 3x - 10) = 0$</p> <p>$-4x^2(x+5)(x-2) = 0$</p> <p>$x = 0$ (mult. 2), -5, 2</p>
<p>5.) $h(x) = x^5 - 81x^3$</p> <p>$x^3(x^2 - 81) = 0$</p> <p>$x^3(x+9)(x-9) = 0$</p> <p>$x = 0$ (mult. 3), -9, 9</p>	<p>6.) $f(x) = 6x^3 + 7x^2 - 5x$</p> <p>$x(6x^2 + 7x - 5) = 0$</p> <p>$x(6x^2 + 10x - 3x - 5) = 0$</p> <p>$x(2x(3x+5) - 1(3x+5)) = 0$</p> <p>$x(2x-1)(3x+5) = 0$</p> <p>$x = 0, \frac{1}{2}, -\frac{5}{3}$</p>

Write a polynomial function in **standard form** with the given zeros.

7.) $x = 0$ (mult. 3) and 6 (mult. 2)

$$x^3(x-6)^2 = 0$$

$$x^3(x^2 - 12x + 36) = 0$$

$$x^5 - 12x^4 + 36x^3 = 0$$

8.) $x = 5, -3,$ and $\frac{1}{4}$

$$(x-5)(x+3)(4x-1) = 0$$

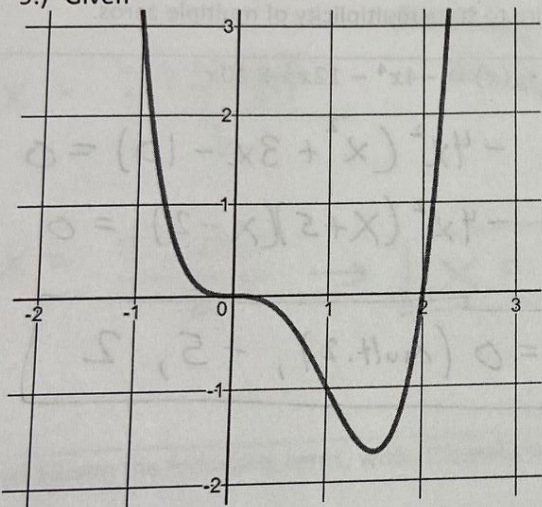
$$(x^2 - 2x - 15)(4x - 1) = 0$$

$$4x^3 - 8x^2 - 60x - x^2 + 2x + 15 = 0$$

$$4x^3 - 9x^2 - 58x + 15 = 0$$

Given the following graphs of polynomial functions, find the following unknowns.

9.) Given



Degree is EVEN or ODD (circle one)

Sign of the Leading Coefficient is positive

Real Zeros are at: 0 and 2

Do any zeros have multiplicity? If so, which ones and how can you tell?

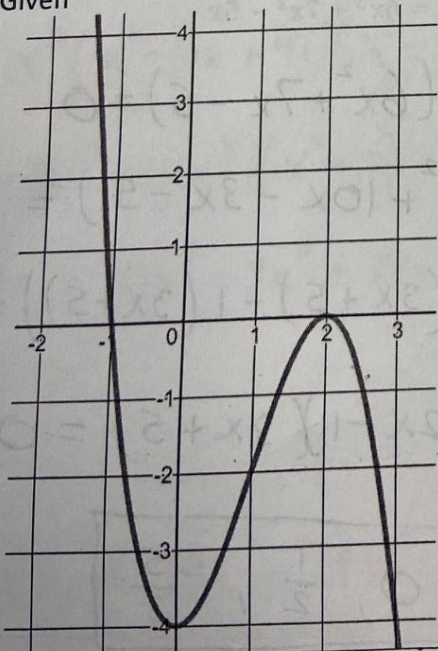
The zero of $x=0$ has

mult. 3 because the

graph "snakes-through" at

that point.

10.) Given



Degree is EVEN or ODD (circle one)

Sign of the Leading Coefficient is negative

Real Zeros are at: -1 and 2

Do any zeros have multiplicity? If so, which ones and how can you tell?

The zero of $x=2$ has

mult. 2 because the graph

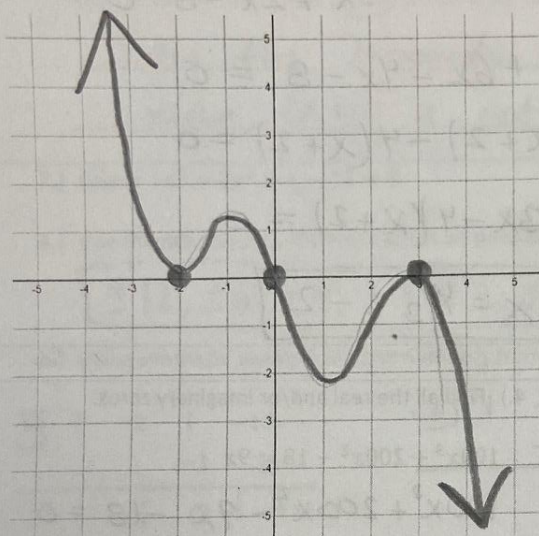
"bounces-back" at that point.

Find the zeros of each function. State multiplicity of multiple zeros. Then sketch a graph the function.

11.) $f(x) = -x(x+2)^2(x-3)^2$

Zeros are at: $X=0, -2$ (mult. 2),
 3 (mult. 2).

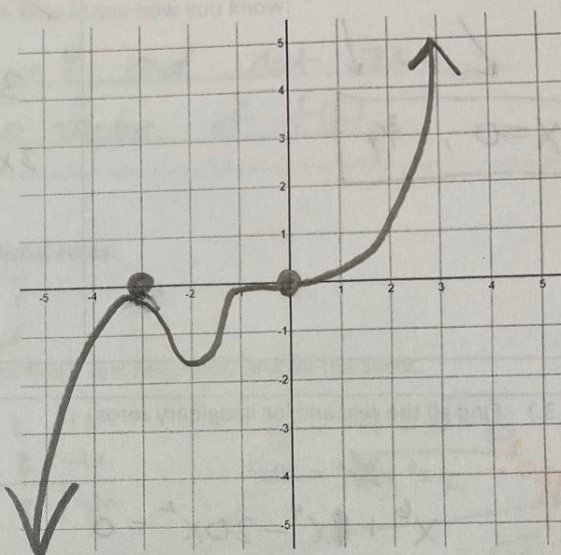
End Behavior: Up / Down



12.) $f(x) = x^3(x+3)^2$

Zeros are at: $X=0$ (mult. 3),
 -3 (mult. 2).

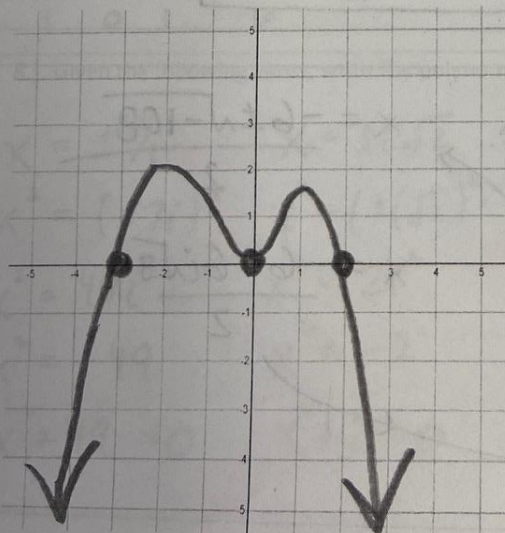
End Behavior: Down / Up



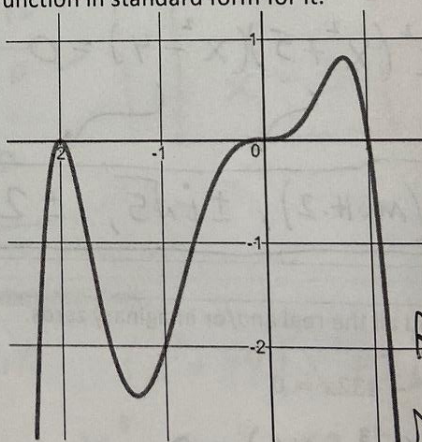
13.) $f(x) = -2x^4 - 2x^3 + 12x^2 - 2x^2(x^2+x-6)$

Zeros are at: $X=-3, 2,$
 0 (mult. 2).

End Behavior: Down / Down



14.) Given the following graph, write a polynomial function in standard form for it.



Right-Side
is
Down " - " a

$$x^3 \cdot (x-2)^2 \cdot (x+1)$$

$$x^3(x^2-4x+4)(x+1) \rightarrow (x^5-4x^4+4x^3)(x)$$

Function in Standard Form:

$$x^6 - 4x^5 + 4x^3 = f(x)$$

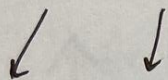
Part 5.2 - Solving Polynomial Equations with Real/Imaginary Zeros -

Directions: Complete the following problems and any zeros should be in simplified form. NO DECIMAL ANSWERS!!! Also, state multiplicity of any multiple zeros. Circle final answers.

1.) Find all the real and/or imaginary zeros.

$$24x^2 - 18x = 0$$

$$6x(4x - 3) = 0$$



$$X = 0, \frac{3}{4}$$

2.) Find all the real and/or imaginary zeros.

$$\frac{6}{2} \frac{-24}{-4}$$

$$3x^2 - 8 = -2x$$

$$3x^2 + 2x - 8 = 0$$

$$3x^2 + 6x - 4x - 8 = 0$$

$$3x(x+2) - 4(x+2) = 0$$

$$(3x - 4)(x + 2) = 0$$

$$X = \frac{4}{3}, -2$$

3.) Find all the real and/or imaginary zeros.

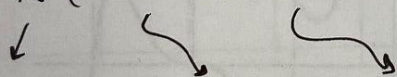
Typo!

$$x^6 + x^4 = 20x^2$$

$$x^6 + x^4 - 20x^2 = 0$$

$$x^2(x^4 + x^2 - 20) = 0$$

$$x^2(x^2 + 5)(x^2 - 4) = 0$$



$$X = 0 (\text{mult. } 2), \pm i\sqrt{5}, \pm 2$$

4.) Find all the real and/or imaginary zeros.

$$100x^3 + 200x^2 - 18 = 9x$$

$$100x^3 + 200x^2 - 9x - 18 = 0$$

$$100x^2(x+2) - 9(x+2) = 0$$

$$(100x^2 - 9)(x+2) = 0$$

$$(10x+3)(10x-3)(x+2) = 0$$

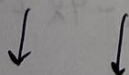
$$X = \pm \frac{3}{10}, -2$$

5.) Find all the real and/or imaginary zeros.

$$2x^4 - 432x = 0$$

$$2x(x^3 - 216) = 0$$

$$2x(x-6)(x^2+6x+36) = 0$$

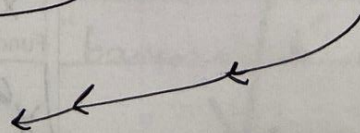


$$X = 0, 6, -3 \pm 3i\sqrt{3}$$

Quadratic Formula!

$$x = \frac{-6 \pm \sqrt{-108}}{2}$$

$$x = \frac{-6 \pm 6i\sqrt{3}}{2}$$



6.) Given the following polynomial function. Determine if $(x + 1)$ is a factor of it.

$$f(x) = x^3 + 6x^2 - 2x - 11$$

$$\begin{array}{r|rrrr} -1 & 1 & 6 & -2 & -11 \\ & & -1 & -5 & 7 \\ \hline & 1 & 5 & -7 & -4 \end{array}$$

Is $(x + 1)$ a factor? YES / **NO** (circle one). Explain here below how you know:

Since the remainder is -4 and not zero,
then $(x+1)$ is not a factor of $f(x)$.

7.) Given: $x^3 - 2x^2 + x - 12 = 0$

a.) Use the rational zero theorem to list all possible rational zeros.

$$\boxed{\pm 12, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1}$$

b.) Using synthetic division and the numbers from above find a real zero, then find all the zeros.

$$\begin{array}{r|rrrr} 1 & 1 & -2 & 1 & -12 \\ & & 1 & -1 & 0 \\ \hline & 1 & -1 & 0 & -12 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & -2 & 1 & -12 \\ & & -2 & 8 & -18 \\ \hline & 1 & -4 & 9 & -30 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & 1 & -12 \\ & & -1 & 3 & -4 \\ \hline & 1 & -3 & 4 & -16 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & -2 & 1 & -12 \\ & & 3 & 3 & 12 \\ \hline & 1 & 1 & 4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 1 & -12 \\ & & 2 & 0 & 2 \\ \hline & 1 & 0 & 1 & -10 \end{array}$$

$$x^2 + x + 4 = 0$$

$$x = \frac{-1 \pm \sqrt{-15}}{2}$$

$$x = \frac{-1 \pm i\sqrt{15}}{2}$$

$$x = 3$$

8.) Given the following zeros, write the polynomial function in standard form.

$$x = \pm 7i$$

$$x = \pm i\sqrt{2}$$

$7i$ and $-i\sqrt{2}$

$$(x^2 + 49)(x^2 + 2) = 0$$

$$x^2 = (\pm 7i)^2$$

$$x^2 = (\pm i\sqrt{2})^2$$

$$x^4 + 2x^2 + 49x^2 + 98 = 0$$

$$x^2 = 49i^2$$

$$x^2 = 2i^2$$

$$x^2 = -49$$

$$x^2 = -2$$

$$x^2 + 49 = 0$$

$$x^2 + 2 = 0$$

$$\boxed{x^4 + 51x^2 + 98 = 0}$$

9.) Find all the real and/or imaginary zeros. Given this graph if the function below.

$$x^4 + 10x^2 = 51 - 40x$$

$$x^4 + 10x^2 + 40x - 51 = 0$$

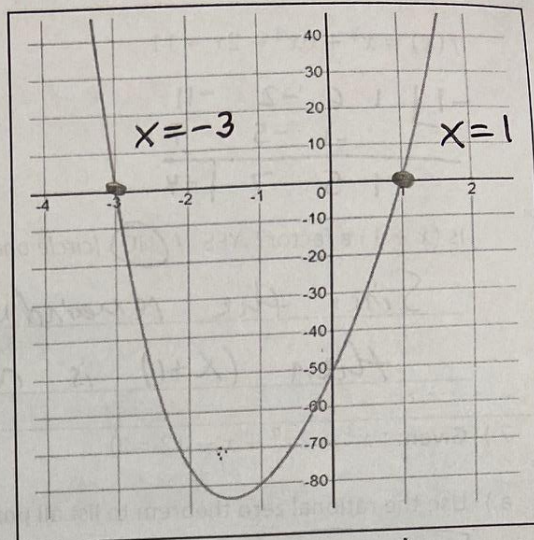
$$\begin{array}{r|rrrrr} -3 & 1 & 0 & 10 & 40 & -51 \\ & & -3 & 9 & -57 & 51 \\ \hline & 1 & -3 & 19 & -17 & 0 \\ & & & 1 & -2 & 17 \\ \hline & 1 & -2 & 17 & 0 & \end{array}$$

$$x^2 - 2x + 17 = 0$$

$$x = \frac{2 \pm \sqrt{-64}}{2}$$

Quadratic Formula!

$$x = \frac{2 \pm 8i}{2} \rightarrow \boxed{x = 1 \pm 4i} \text{ and } \boxed{x = -3, 1}$$



10.) Given the following zeros, write the polynomial function in standard form.

0 (mult. 3) and $4 + 5i$ and $\sqrt{3}$

$$x^3(x^2 - 8x + 41)(x^2 - 3) = 0$$

$$x = 4 \pm 5i$$

$$x - 4 = \pm 5i$$

$$(x - 4)^2 = (\pm 5i)^2$$

$$(x^5 - 8x^4 + 41x^3)(x^2 - 3) = 0$$

$$x^7 - 8x^6 + 41x^5 - 3x^5 + 24x^4 - 123x^3$$

$$x^2 - 8x + 16 = -25$$

$$x^2 - 8x + 41 = 0$$

$$x = \pm\sqrt{3}$$

$$x^2 = (\pm\sqrt{3})^2$$

$$x^2 = 3$$

$$x^2 - 3 = 0$$

$$\boxed{x^7 - 8x^6 + 38x^5 + 24x^4 - 123x^3 = 0}$$