Topic 1.1 - Day 3: Operations w/ Functions

The Algebra of Functions: Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions. The **sum** $f + g$, the **difference** $f - g$, the **product** fg, and the **quotient** $\frac{f}{g}$ are functions whose domains are the set of all real numbers common to the domains of f and g:

1. Sum: $(f + g)(x) = f(x) + g(x)$
2. Difference: $(f - g)(x) = f(x) - g(x)$
3. Product: $(fg)(x) = f(x) \cdot g(x)$
4. Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$.

Example:

For each function, $f(x) = 2x$ and $g(x) = x - 1$.

Sum: $f+g$

$$(f + g)(x) = f(x) + g(x)$$
$$= 2x + (x - 1) = \boxed{3x - 1}$$

Difference: $f-g$

$$(f - g)(x) = f(x) - g(x)$$
$$= 2x - (x - 1) = 2x - x + 1 = \boxed{x + 1}$$

Product: fg

$$(fg)(x) = f(x) \cdot g(x)$$
$$= 2x(x - 1) = \boxed{2x^2 - 2x}$$

Quotient: $\frac{f}{g}$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \boxed{\frac{2x}{x - 1}} \quad x \neq 1$$

Ex2.

Let $f(x) = x + 3$ *and* $g(x) = x^2 - 2x - 15$. *Find each of the following functions and the domains:*

$$a.\ (f + g)(x) \qquad b.\ (f - g)(x) \qquad c.\ (fg)(x) \qquad d.\ \left(\frac{f}{g}\right)(x)$$

a.) $(x+3)+(x^2-2x-15)$

x^2-x-12

D: $(-\infty,\infty)$

b.) $(x+3)-(x^2-2x-15)$

$-x^2+3x+18$

D: $(-\infty,\infty)$

c.) $(x+3)\cdot(x^2-2x-15)$

$x^3-2x^2-15x+3x^2-6x-45$

$x^3+x^2-21x-45$ D: $(-\infty,\infty)$

d.) $\frac{(x+3)}{(x-5)(x+3)} \rightarrow \frac{1}{x-5}$

D: $(-\infty,-3)\cup(-3,5)\cup(5,\infty)$

The Composition of Functions

The **composition of the function f with g** is denoted by $f \circ g$ and is defined by the equation

$$(f \circ g)(x) = f(g(x)).$$

g(x) "inside" of f(x)

Ex3.

Let $f(x) = \sqrt{x}$ *and* $g(x) = x^2 - 5$ *and* $h(x) = x + 3$. *Find each of the following:*

a. $(f \circ h)(x)$ *b.* $(g \circ h)(x)$ *c.* $(h \circ f)(25)$ *d.* $(h \circ f \circ g)(3)$

a. $f(h(x))$ → $f(x+3)$ → $\sqrt{x+3}$

b. $g(h(x))$ → $g(x+3)$ → $(x+3)^2-5$ → $x^2+6x+9-5$ → x^2+6x+4

c. $h(f(25))$ → $h(5)$ → 8

d. $h(f(g(3)))$ → $h(f(4))$ → $h(2)$ → 5