Topic 1.2: Inverse Functions - Day 1

Verifying and Finding Inverse Functions

Day 1 --- HW Exercise Set 1.8
#'s:
1 - 5 odds, 15 - 25 odds

Definition of the Inverse of a Function

Let f and g be two functions such that

$$f(g(x)) = x \qquad \text{for every } x \text{ in the domain of } g$$

and

$$g(f(x)) = x \qquad \text{for every } x \text{ in the domain of } f.$$

The function g is the **inverse of the function** f and is denoted by f^{-1} (read "f-inverse"). Thus, $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. The domain of f is equal to the range of f^{-1}, and vice versa.

GREAT QUESTION!

Is the -1 in f^{-1} an exponent?

The notation f^{-1} represents the inverse function of f. The -1 is *not* an exponent. The notation f^{-1} does *not* mean $\frac{1}{f}$:

$$f^{-1} \neq \frac{1}{f}.$$

Ex1. **Verify** that the given functions are inverses of each other.

$$f(x) = 3x + 2 \quad \text{and} \quad g(x) = \frac{x-2}{3}$$

$$f(g(x)) \rightarrow f\left(\frac{x-2}{3}\right) \rightarrow \cancel{3}\left(\frac{x-2}{\cancel{3}}\right)+2 \rightarrow (x-2)+2 = \boxed{x}$$

$$g(f(x)) \rightarrow g(3x+2) \rightarrow \frac{(3x+2)-2}{3} \rightarrow \frac{(3x)}{3} = \boxed{x}$$

$$\boxed{f(x) \text{ and } g(x) \text{ are inverses of eachother}}$$

How to find an inverse...

Switch the x and y values!

$f(x)$	
x	y
-1	1
0	4
1	5
2	-1

$\longleftrightarrow$

$f^{-1}(x)$	
x	y
1	-1
4	0
5	1
-1	2

Finding the Inverse of a Function

The equation for the inverse of a function f can be found as follows:

1. Replace $f(x)$ with y in the equation for $f(x)$.
2. Interchange x and y.
3. Solve for y. If this equation does not define y as a function of x, the function f does not have an inverse function and this procedure ends. If this equation does define y as a function of x, the function f has an inverse function.
4. If f has an inverse function, replace y in step 3 by $f^{-1}(x)$. We can verify our result by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

This is called the "*switch and solve*" strategy. We switch *x* and *y*, then solve for *y*.

Ex2. Find the equation for $f^{-1}(x)$, the inverse function.

a. $f(x) = x^3 + 1$

$y = x^3 + 1$

$x = y^3 + 1$

$\sqrt[3]{x-1} = \sqrt[3]{y^3}$

$\sqrt[3]{x-1} = y$

$\boxed{f^{-1}(x) = \sqrt[3]{x-1}}$

b. $f(x) = \sqrt{x - 1}$

$y = \sqrt{x-1}$

$x^2 = \sqrt{y-1}^{\,2}$

$x^2 = y - 1$

$x^2 + 1 = y$

$\boxed{f^{-1}(x) = x^2 + 1}$

c. $f(x) = \dfrac{5}{x} + 4$

$y = \dfrac{5}{x} + 4$

$x = \dfrac{5}{y} + 4$

$y(x-4) = \dfrac{5}{\cancel{y}} \cdot \cancel{y}$

$y \cdot (x-4) = 5$

$y = \dfrac{5}{(x-4)}$

$\boxed{f^{-1}(x) = \dfrac{5}{(x-4)}}$