

Topic 1.2: Inverse Functions - Day 2

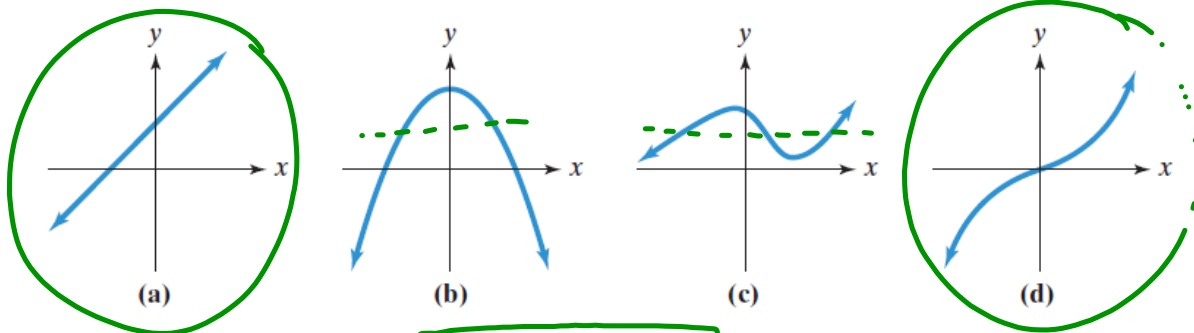
Graphs and Inverse Functions

Day 2 --- HW Exercise Set 1.8
#s:
29 - 37 all, 39, 41, 42, 45

The Horizontal Line Test for Inverse Functions

A function f has an inverse that is a function, f^{-1} , if there is no horizontal line that intersects the graph of the function f at more than one point.

Ex1. Which graphs represent functions that have inverse functions?



Such a function is called a **one-to-one function**. Thus, a **one-to-one function is a function in which no two different ordered pairs have the same second component. Only one-to-one functions have inverse functions.** Any function that passes the horizontal line test is a one-to-one function. Any one-to-one function has a graph that passes the horizontal line test.

Graphs of f and f^{-1}

There is a relationship between the graph of a one-to-one function, f , and its inverse, f^{-1} . Because inverse functions have ordered pairs with the coordinates interchanged, if the point (a, b) is on the graph of f , then the point (b, a) is on the graph of f^{-1} . The points (a, b) and (b, a) are symmetric with respect to the line $y = x$. Thus, **the graph of f^{-1} is a reflection of the graph of f about the line $y = x$.** This is illustrated in Figure 1.68.

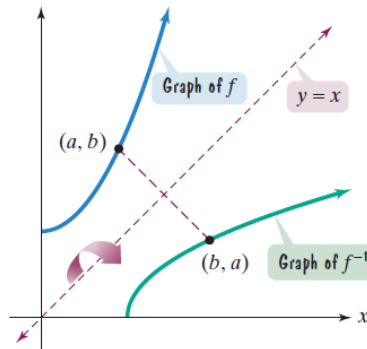
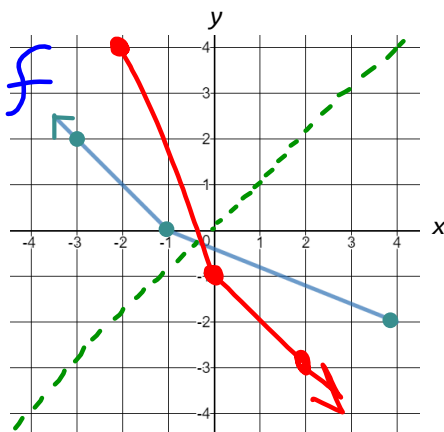


FIGURE 1.68 The graph of f^{-1} is a reflection of the graph of f about $y = x$.

Ex2. Use the graph of f to draw the graph of its inverse f^{-1} .



f
(4, -2)
(-1, 0)
(-3, 2)

f⁻¹
(-2, 4)
(0, -1)
(2, -3)

Ex3. Do each of the following:

- Graph f and f^{-1} in the same coordinate plane.
- Find the domain and range of f and f^{-1} .
- Find an equation for $f^{-1}(x)$.

$$f(x) = 2x - 3$$

slope *y-int*

$$f: \underline{D}: (-\infty, \infty)$$

$$\underline{R}: (-\infty, \infty)$$

$$f^{-1}: \underline{D}: (-\infty, \infty)$$

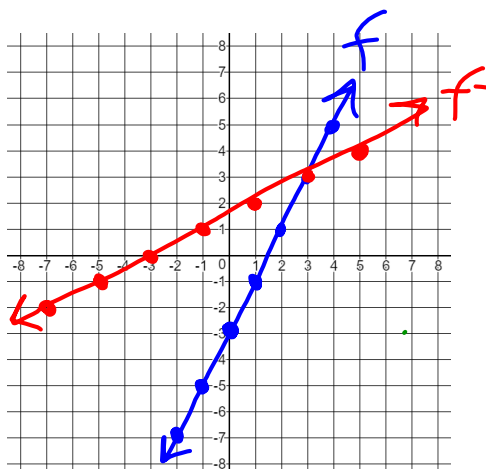
$$\underline{R}: (-\infty, \infty)$$

$$y = 2x - 3$$

$$x = 2y - 3 \rightarrow \frac{1}{2}x + \frac{3}{2} = y$$

$$\frac{x+3}{2} = \frac{2y}{2}$$

$$f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$$



Ex3. Do each of the following:

- Graph f and f^{-1} in the same coordinate plane.
- Find the domain and range of f and f^{-1} .
- Find an equation for $f^{-1}(x)$.

$$f(x) = x^2 - 1, \text{ if } x \geq 0$$

$$f: \underline{D}: [0, \infty)$$

$$\underline{R}: [-1, \infty)$$

$$f^{-1}: \underline{D}: [-1, \infty)$$

$$\underline{R}: [0, \infty)$$

$$f(x) = x^2 - 1$$

$$y = x^2 - 1$$

$$x = y^2 - 1$$

$$\sqrt{x+1} = \sqrt{y^2}$$

$$f^{-1}(x) = \sqrt{x+1}$$

