

Topic 1.3: Transformations of Functions

Day 1 - Performing Transformations on Graphs

I. Vertical and Horizontal Shifts (Translations)

Given a function, $y = f(x)$ and c , as a real number,

A **vertical shift up**, when $y = f(x) + c$.

A **vertical shift down**, when $y = f(x) - c$.

} A number being added or subtracted, **outside** of the function.

A **horizontal shift right**, when $y = f(x - c)$.

A **horizontal shift left**, when $y = f(x + c)$.

} A number being added or subtracted, **inside** of the function.

Ex1. Given the graph of $y = f(x)$, graph the following transformation functions below.

a. $g(x) = f(x) - 5$

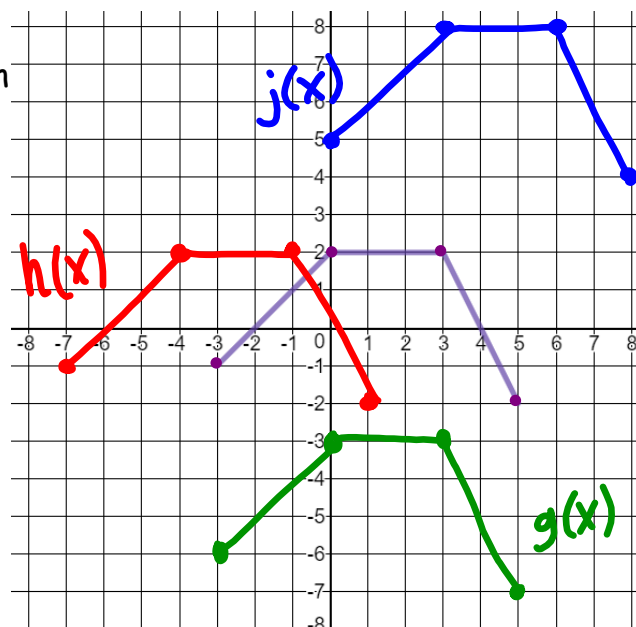
Down 5

b. $h(x) = f(x + 4)$

Left 4.

c. $j(x) = f(x - 3) + 6$

Right 3, up 6



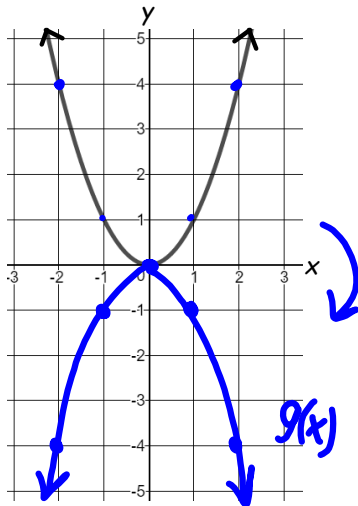
II. Reflections

Given a function, $y = f(x)$,

A reflection about the x-axis,

when $y = -f(x)$.

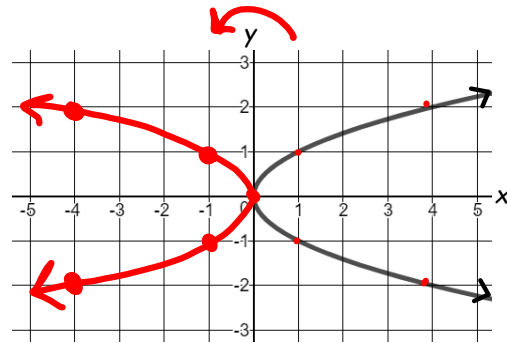
A negative value, outside
of the function.



A reflection about the y-axis,

when $y = f(-x)$.

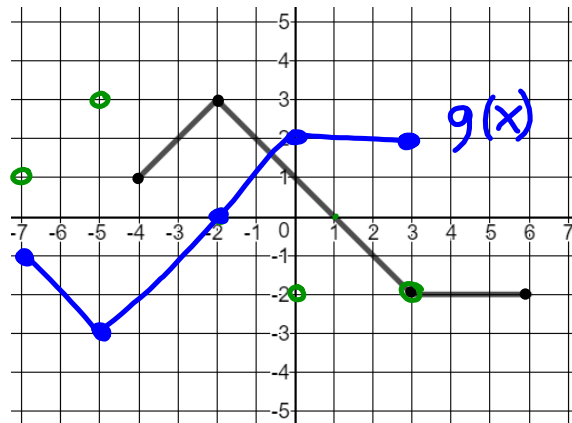
A negative value, inside
of the function.



Ex2. Given the graph of $y = f(x)$, graph the following transformation functions below.

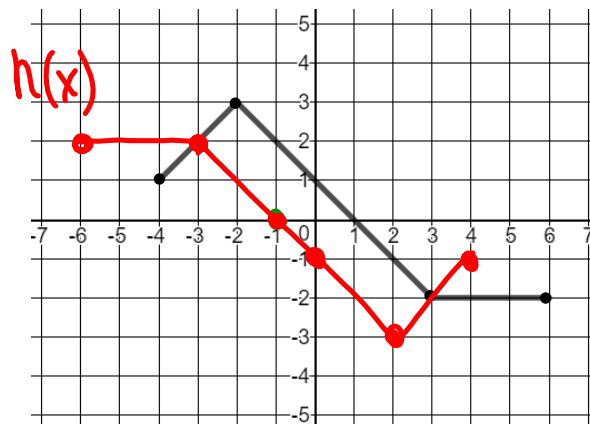
a. $g(x) = -f(x + 3)$

1. Left + 3
2. reflect over x-axis.



b. $h(x) = -f(-x)$

1. Reflect over y-axis.
2. Reflect over x-axis.



III. Vertical and Horizontal Stretching/Shrinking (Dilations)

Given a function, $y = f(x)$,

VERTICAL: Multiply your y -values by this amount

A **vertical stretch**,

when $y = \underbrace{c}_{> 1} f(x)$.

When $c > 1$, outside of the function.

A **vertical shrink**,

when $y = \underbrace{c}_{< 1} f(x)$.

When $0 < c < 1$, inside of the function.

HORIZONTAL: Divide your x -values by this amount

*A **horizontal stretch**,

when $y = f(\underbrace{c}_{< 1}x)$.

When $0 < c < 1$, inside of the function.

*A **horizontal shrink**,

when $y = f(\underbrace{c}_{> 1}x)$.

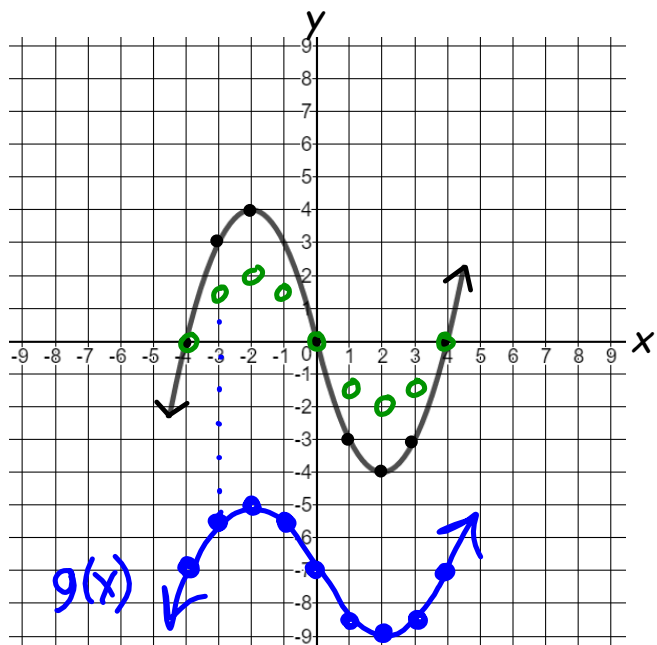
When $c > 1$, inside of the function.

Ex3. Given the graph of $y = f(x)$, graph the following transformation function below.

a. $g(x) = 1/2f(x) - 7$

1. Vertical shrink by $\frac{1}{2}$.

2. Down 7.



A function involving more than one transformation can be graphed by performing transformations in the following order:

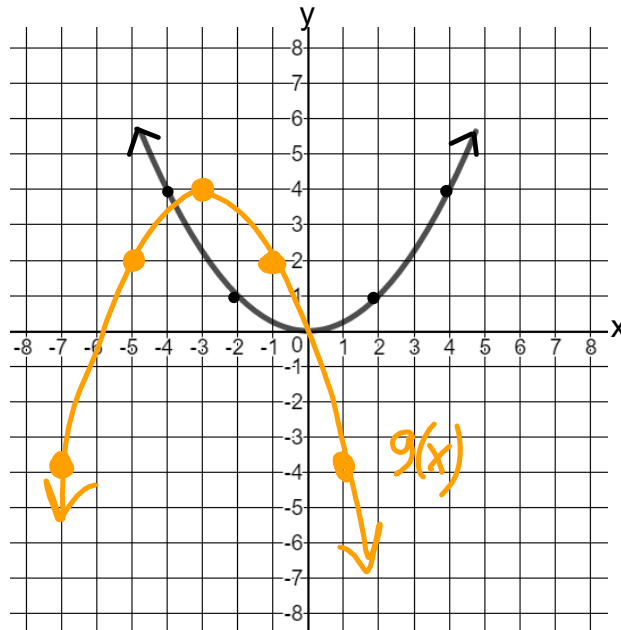
1. Horizontal shifting
2. Stretching or shrinking
3. Reflecting
4. Vertical shifting

IV. Combination of Transformations

Ex4. Given the graph of $y = f(x)$, graph the following transformation function below.

$$g(x) = -2f(x + 3) + 4$$

1. Left 3.
2. Vertical stretch by 2.
3. reflect over x-axis.
4. Up 4.



Ex5. Given the graph of $y = f(x)$, graph the following transformation function below.

$$g(x) = 1/2f(-x) - 5$$

1. Vertical Shrink $1/2$.
2. Reflect over y-axis
3. Down 5.

