Topic #4: Rational Functions

Topic 1.4 - Day 1 - Properties of Rational Functions

Rational functions are quotients of polynomial functions. This means that rational functions can be expressed as

$$f(x) = \frac{p(x)}{q(x)},$$

Day 1 --- HW Exercise Set 2.6 #'s: 1 - 7 odds, 21 - 43 odds

where p and q are polynomial functions and $q(x) \neq 0$. The **domain** of a rational function is the set of all real numbers except the x-values that make the denominator zero. For example, the domain of the rational function

$$f(x) = \frac{x^2 + 7x + 9}{x(x-2)(x+5)}$$

This is $p(x)$.

This is $q(x)$.

is the set of all real numbers except 0, 2, and -5.

Ex1. Find the domain of each rational function.

$$f(x) = \frac{5}{x^2 - 6x}$$

$$\frac{5}{x(x-6)}$$

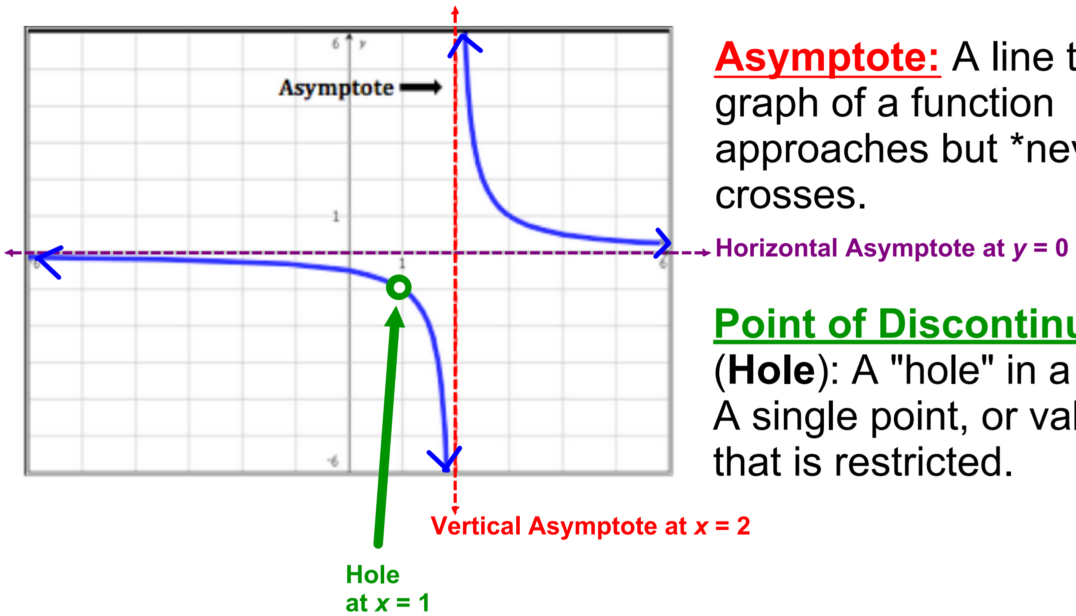
$x \neq 0$ $\quad$ $x \neq 6$

$$\boxed{(-\infty,0)\cup(0,6)\cup(6,\infty)}$$

$$f(x) = \frac{x+3}{x^2 - 64}$$

$$\frac{x+3}{(x+8)(x-8)}$$

$$\boxed{(-\infty,-8)\cup(-8,8)\cup(8,\infty)}$$



Asymptote: A line that graph of a function approaches but *never crosses.

Point of Discontinuity (**Hole**): A "hole" in a graph. A single point, or value of x, that is restricted.

How to find the vertical asymptotes and holes in a rational function:

1. Factor first.
2. Find the restricted values of *x* in the denominator.
3. Simplify/Cancel. Any factors of *x* that cancel from the denominator is a **hole** in the graph.
4. Factors of x that are left over in the denominator are the **vertical asymptotes.**

Ex3.

$$f(x)=\frac{5}{x^2+6x} \qquad f(x)=\frac{x-3}{x^2-9} \qquad f(x)=\frac{x^2+x-12}{x^2-3x}$$

$$\frac{5}{x(x+6)}$$

VA: $x=0$, $x=-6$

$$\frac{\cancel{(x-3)}}{(x+3)\cancel{(x-3)}}$$

Hole: $x=3$

VA: $x=-3$

$$\frac{\cancel{(x-3)}(x+4)}{x\cancel{(x-3)}}$$

Hole: $x=3$

VA: $x=0$

Finding Horizontal asymptotes:

Lets let, **n** = degree of numerator
d = degree of denominator

- If **n<d**; the horizontal asymptote is **y = 0.** ex. $\frac{x}{x^2+3}$
- If **n>d**; there is **NO** horizontal asymptote. ex. $\frac{x^2+5}{4x}$
- If **n=d**; the horizontal asymptote is $y = \frac{a}{b}$ ex. $\frac{3x^1+1}{1x^1-2}$

*Where **a** and **b** are the leading coefficients of the numerator and denominator.*

$y=3$

Ex4.

What is the horizontal asymptote for the rational function?

a. $y=\frac{-2x+6}{1x-5}$ **b.** $y=\frac{x-1}{x^2+4x+4}$ **c.** $y=\frac{x^2+2x-3}{x-2}$

$\boxed{HA: y=-2}$ $\boxed{HA: y=0}$ $\boxed{HA: \text{none}}$