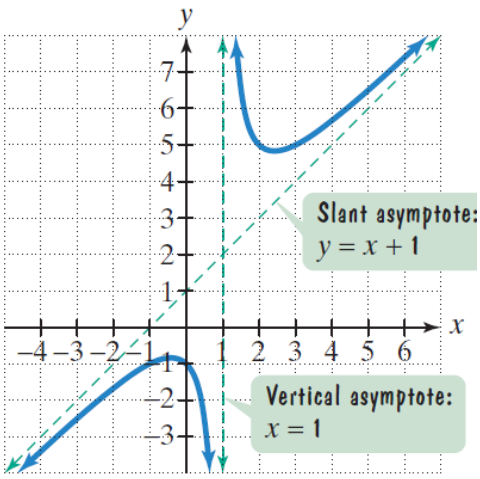
Topic 1.4 - Day 3 - Slant Asymptotes

Examine the graph of

$$f(x) = \frac{x^2 + 1}{x - 1},$$

The graph of a rational function has a slant asymptote if the degree of the numerator is one more than the degree of the denominator. The equation of the slant asymptote can be found by division. For example, to find the slant asymptote for the graph of $f(x) = \frac{x^2 + 1}{x - 1}$, divide $x - 1$ into $x^2 + 1$:



Do you remember "synthetic division" ?

$x^2 + 1$ divided by $x - 1$

In general, if $f(x) = \frac{p(x)}{q(x)}$, p and q have no common factors, and the degree of p is one greater than the degree of q, find the slant asymptote by dividing $q(x)$ into $p(x)$. The division will take the form

$$\frac{p(x)}{q(x)} = mx + b + \frac{\text{remainder}}{q(x)}.$$

Slant asymptote: $y = mx + b$

The equation of the slant asymptote is obtained by dropping the term with the remainder. Thus, the equation of the slant asymptote is $y = mx + b$.

Ex1. Find the slant asymptotes of these rational functions.

$$f(x) = \frac{2x^2 - 5x + 7}{x - 2}$$

2 | 2 -5 7
4 -2
2 -1 5

remainder

$y = 2x - 1$

$$f(x) = \frac{x^2 - 3}{x + 1}$$

-1 | 1 0 -3
-1 1
1 -1 -2

$y = 1x - 1$

Ex2. Find all unknowns and graph the rational function.

$$f(x) = \frac{x^2 - 4x - 5}{x - 3}$$

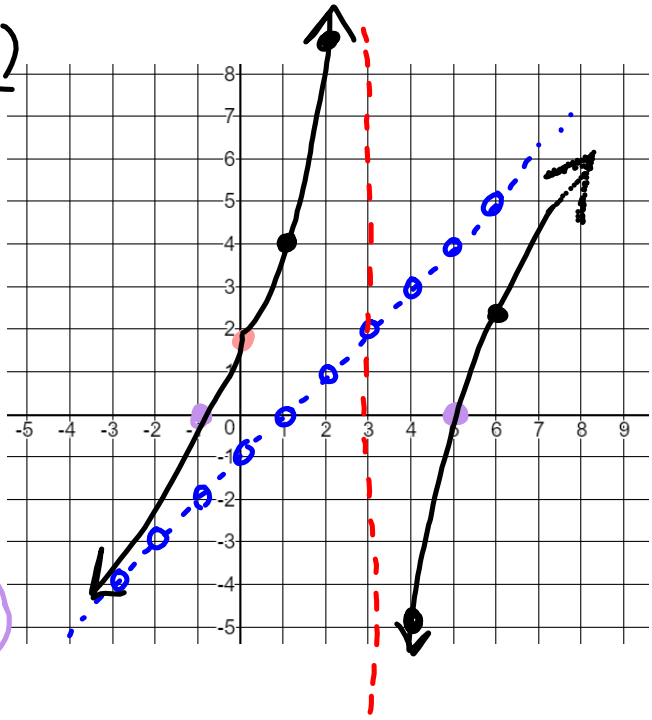
$\frac{(x-5)(x+1)}{(x-3)}$

3 | 1 -4 -5
3 -3
1 -1 | -8

VA: $x = 3$

SA: $y = 1x - 1$

- y-int: $\frac{-5}{-3} = 1.67$
- x-int: $x^2 - 4x - 5 = 0$ → 5, -1
  $(x-5)(x+1) = 0$



Ex3. Find all unknowns and graph the rational function.

$$f(x) = \frac{x^2 + 1}{x + 2}$$

- x-int: $x^2 + 1 = 0$, $x^2 = -1$, $x = i$, none

-2 | 1 0 1
-2 4
1 -2 | 5

- SA: $y = 1x - 2$

VA: $x = -2$

- y-int: $\frac{1}{2} \rightarrow 0.5$