10.1 - Counting Principles

Fundamental Counting Principle: If event *M* can occur in *m* ways and is followed by event *N* that can occur *n* ways, then event *M* followed by event *N* can occur in *nm* total ways.

Example1: A local deli has seven different types of sandwiches they serve, as well as, three types of soups, and six beverages. If a lunch special comes with a sandwich, a soup and a beverage, how many possible unique lunch specials can be made?

$7 \cdot 3 \cdot 6 =$ 126 unique specials

Example2: A code for a digital safe must be set. You must use 4 single digit numbers first (which can be repeated), followed by 3 letters of the alphabet (no letters may be repeated). How many possible unique ways can this safe code be set?

$10 \cdot 10 \cdot 10 \cdot 10 \cdot 26 \cdot 25 \cdot 24$

156,000,000 unique codes

Permutation: an arrangement in a definite order of a number of items, taken some or all at a time.

ORDER MATTERS!

In a permutation : ABC $\neq$ CBA

Example3: I have seven colored markers. How many unique ways can I arrange four of the markers on the table?

$7 \cdot 6 \cdot 5 \cdot 4 = 840$

There is also a formula for permutations. The number of unique ways you can arrange *n* items, *r* items at a time: $_n\mathrm{Pr} = \frac{n!}{(n-r)!}$

$_7P_4 = \frac{7!}{3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 840$

Permutation: $nPr = \frac{n!}{(n-r)!}$

Example4: 15 students are in a race. First, second and third place will win gold, silver and bronze medals respectively. How many possible ways could the runners finish first, second and third?

Questions you could ask yourself....

1. Does the order matter? Yes
2. Up to how many places? 3
3. As far as awards go, does it matter if the students finish 4 - 15th places? No

$${}_{15}P_3 = \frac{15!}{12!} = 15 \cdot 14 \cdot 13 = \boxed{2730}$$

Combination: a selection of items in which the order is not regarded and repetition is allowed.

ORDER DOES NOT MATTER!

In a combination : ABC = CBA

There is also a formula for combinations. The number of unique ways you can select *n* items, *r* items at a time, without regard to order is:

$$nCr = \frac{n!}{r!(n-r)!}$$

Example5: There are 5 students in a class that want to go on a field trip to Six Flags Great America, but there are only three tickets available. How many different groups of three students could go?

$${}_5C_3 = \frac{5!}{3!\,2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = \frac{20}{2} = \boxed{10}$$

Combination or Permutation ???

1.) Your History teacher assigned each student in her class of 28 students to design a map of the world. She then intends to select 4 of them to put on display in the hallway. How many different ways can she select the 4 maps?

$$_{28}C_4 = \frac{28!}{4!\,24!} = \frac{28\cdot 27\cdot 26\cdot 25}{4\cdot 3\cdot 2\cdot 1} = 20{,}475$$

2.) You enter a raffle drawing in which 100 raffle tickets are sold. The first ticket drawn wins an iPad, the second wins $100, and the third wins $25. How many different ways can three winning tickets be drawn?

$$_{100}P_3 = \frac{100!}{97!} = 100\cdot 99\cdot 98 = 970{,}200$$