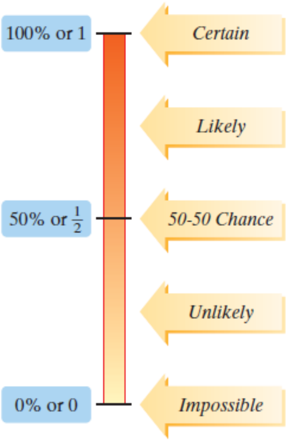
10.2 - Probability - Day 1

Probabilities of events are expressed as numbers ranging from 0 to 1, or 0% to 100%. The closer the probability of a given event is to 1, the more likely it is that the event will occur. The closer the probability of a given event is to 0, the less likely it is that the event will occur.



Possible Values for Probabilities

Empirical Probability

Empirical probability applies to situations in which we observe how frequently an event occurs. We use the following formula to compute the empirical probability of an event:

Computing Empirical Probability

The **empirical probability** of event E, denoted by $P(E)$, is

$$P(E) = \frac{\text{observed number of times } E \text{ occurs}}{\text{total number of observed occurrences}}.$$

HW: 10.2

Day 1: #'s: 1 - 30 all *(29 and 30 are challenge questions)*.

Example 1 - Empirical Probability

	Age		
Gender	Under 40	40 or older	TOT:
Male	12	2	14
Female	8	3	11
TOT	20	5	25

The table to the left shows the distribution of age and gender for 25 people who enter a contest. If the contest winner will be selected at random, what is the probability that......

a.) the winner will be female?

$\boxed{\frac{11}{25}}$ OR $.44 \rightarrow \boxed{44\%}$

b.) the winner will be 40 or older?

$\frac{5}{25} \rightarrow \boxed{\frac{1}{5}}$ OR $.20$ $\boxed{20\%}$

c.) the winner will be a male 40 or older OR a female under 40?

$\frac{2+8}{25} = \frac{10}{25} = \boxed{\frac{2}{5}}$ OR $.40 \rightarrow \boxed{40\%}$

Theoretical Probability

You toss a coin. Although it is equally likely to land either heads up, denoted by H, or tails up, denoted by T, the actual outcome is uncertain. Any occurrence for which the outcome is uncertain is called an **experiment**. Thus, tossing a coin is an example of an experiment. The set of all possible outcomes of an experiment is the **sample space** of the experiment, denoted by S. The sample space for the coin-tossing experiment is

$$S = \{\underbrace{H}_{\text{Lands heads up}}, \underbrace{T}_{\text{Lands tails up}}\}.$$

We can define an event more formally using these concepts. An **event**, denoted by E, is any subcollection, or subset, of a sample space. For example, the subset $E = \{T\}$ is the event of landing tails up when a coin is tossed.

Theoretical probability applies to situations like this, in which the sample space only contains equally likely outcomes, all of which are known. To calculate the theoretical probability of an event, we divide the number of outcomes resulting in the event by the number of outcomes in the sample space.

Computing Theoretical Probability

If an event E has $n(E)$ equally likely outcomes and its sample space S has $n(S)$ equally likely outcomes, the **theoretical probability** of event E, denoted by $P(E)$, is

$$P(E) = \frac{\text{number of outcomes in event } E}{\text{number of outcomes in sample space } S} = \frac{n(E)}{n(S)}.$$

The sum of the theoretical probabilities of all possible outcomes in the sample space is 1.

EXAMPLE 2 Computing Theoretical Probability

FIGURE 10.11 Outcomes when a die is rolled

a.) A die is rolled. Find the probability of getting a two or less

$\frac{2}{6} \rightarrow \boxed{\frac{1}{3}} \rightarrow \boxed{33.3\%}$

b.) One die is rolled. Find the probability of getting a number less than 5.

$\frac{4}{6} \rightarrow \boxed{\frac{2}{3}} \rightarrow \boxed{66.7\%}$

c.) Two die are rolled. Find the probability of getting a sum of 8.

Solution to part c.) from previous slide......

Each die has six equally likely outcomes. By the Fundamental Counting Principle, there are $6 \cdot 6$, or 36, equally likely outcomes in the sample space. That is, $n(S) = 36$. The 36 outcomes are shown below as ordered pairs. The five ways of rolling a sum of 8 appear in the green highlighted diagonal.

First Die \ Second Die	⚀	⚁	⚂	⚃	⚄	⚅
⚀	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
⚁	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
⚂	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
⚃	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
⚄	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
⚅	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

This event has 5 outcomes, so $n(E) = 5$. Thus, the probability of getting a sum of 8 is

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$$ OR $\boxed{14\%}$

A standard 52-card deck of cards

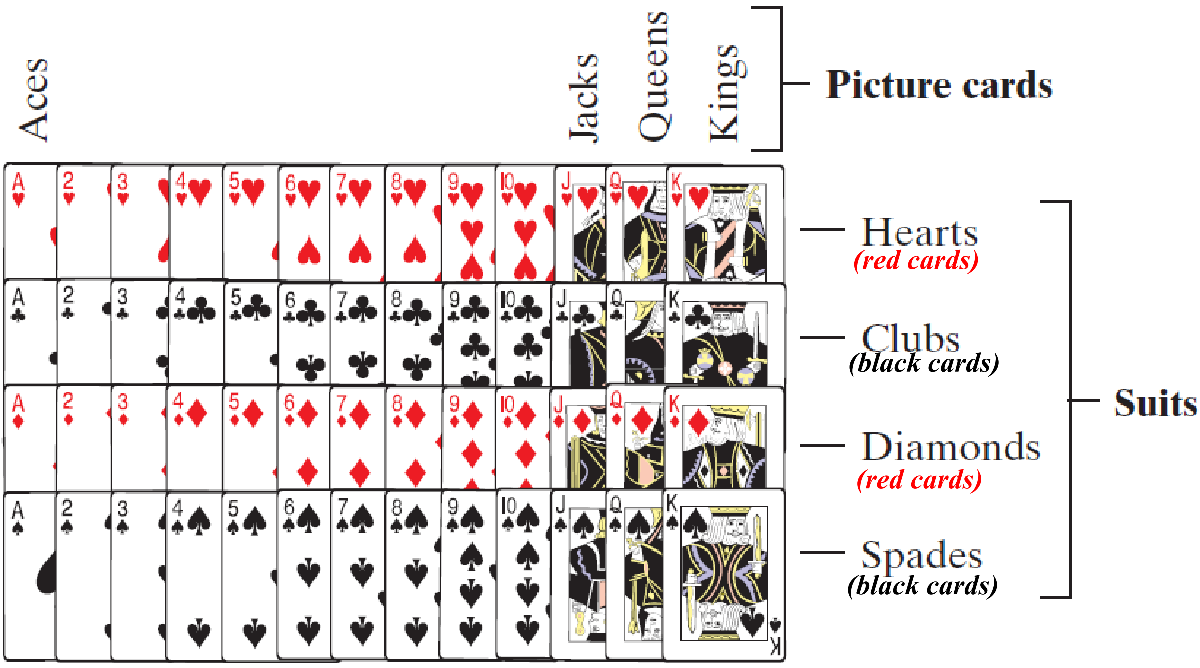


FIGURE 10.12 A standard 52-card bridge deck

EXAMPLE 4 Probability and a Deck of 52 Cards

You are dealt one card from a standard 52-card deck. Find the probability of......

a.) getting a red card?

$\frac{26}{52} \rightarrow \boxed{\frac{1}{2}}$ OR $\boxed{50\%}$

b.) getting a seven?

$\frac{4}{52} \rightarrow \boxed{\frac{1}{13}}$ OR $\boxed{7.7\%}$

c.) getting a face (*picture*) card?

$\frac{12}{52} \rightarrow \boxed{\frac{3}{13}}$ OR $\boxed{23\%}$

d.) getting an even numbered card that is black?

$\frac{10}{52} \rightarrow \boxed{\frac{5}{26}}$ OR $\boxed{19.2\%}$

EXAMPLE 5 Probability and Combinations: Winning the Lottery

Florida's lottery game, LOTTO, is set up so that each player chooses six different numbers from 1 to 53. If the six numbers chosen match the six numbers drawn randomly, the player wins (or shares) the top cash prize. (As of this writing, the top cash prize has ranged from \$7 million to \$106.5 million.) With one LOTTO ticket, what is the probability of winning this prize?

$$\frac{1}{{}_{53}C_6} = \boxed{\frac{1}{22957480}}$$

$$\frac{53!}{6!\,47!} = \frac{(53\cdot 52\cdot 51\cdot 50\cdot 49\cdot 48)}{(6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1)} =$$