10.2 - Probability - Day 3

And Probabilities with Independent Events

Suppose that you toss a fair coin two times in succession. The outcome of the first toss, heads or tails, does not affect what happens when you toss the coin a second time. For example, the occurrence of tails on the first toss does not make tails more likely or less likely to occur on the second toss. The repeated toss of a coin produces *independent events* because the outcome of one toss does not influence the outcome of others. Two events are **independent events** if the occurrence of either of them has no effect on the probability of the other.

If two events are independent, we can calculate the probability of the first occurring and the second occurring by multiplying their probabilities.

And Probabilities with Independent Events

If A and B are independent events, then

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

*** also works for dependent events! You just have to alter the sample space after the first event.***

HW: 10.2 Day 3: #'s: 47 - 53 all, 64 - 71 all

Don't do #69 !

EXAMPLE 10 Independent Events on a Roulette Wheel

Figure 10.18 shows a U.S. roulette wheel that has 38 numbered slots (1 through 36, 0, and 00). Of the 38 compartments, 18 are black, 18 are red, and 2 are green. A play has the dealer spin the wheel and a small ball in opposite directions. As the ball slows to a stop, it can land with equal probability on any one of the 38 numbered slots.

a.) What is the probability of red winning on two consecutive plays?

$$\frac{18}{38} \cdot \frac{18}{38} = \boxed{\frac{324}{1444}} \rightsquigarrow 22.4\%$$

b.) What is the probability of green winning on two consecutive plays?

$$\frac{2}{38} \cdot \frac{2}{38} = \boxed{\frac{4}{1444}} \rightsquigarrow .28\%$$

c.) What is the probability of an even number (*0 and 00 are not even*) winning, and then a multiple of seven winning on two consecutive plays?

$$\frac{18}{38} \cdot \frac{5}{38} = \boxed{\frac{90}{1444}} \rightsquigarrow 6.2\%$$

The *and* rule for independent events can be extended to cover three or more events. Thus, if A, B, and C are independent events, then

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C).$$

EXAMPLE 11 Independent Events

Suppose a six-sided dice is rolled. What is the probability that a four will be rolled five times consecutively?

$$\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \boxed{\frac{1}{7776}} \rightsquigarrow .013\,\%$$

AND Probability with Dependant Events

a.) There are 12 girls and 8 boys in the Math Club. Suppose random people are chosen from the club. What is the probability that the first one randomly chosen is a girl and then the second is a different girl?

$$\frac{12}{20} \cdot \frac{11}{19} = \boxed{\frac{132}{380}} \rightsquigarrow 34.7\,\%$$

b.) Suppose I deal you 5 random cards from a 52-card deck, what is the probability of the first card being an ace, the second an ace, then the third, fourth, and fifth a king?

$$\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{4}{50} \cdot \frac{3}{49} \cdot \frac{2}{48} = \boxed{\frac{288}{311875200}} \rightsquigarrow \boxed{.0000923\,\%}$$