

13-8 Reciprocal Trig. Functions

Essential Understanding Cosine, sine, and tangent have reciprocals. Cosine and *secant* are reciprocals, as are sine and *cosecant*. Tangent and *cotangent* are also reciprocals.

take note

Key Concept Cosecant, Secant, and Cotangent Functions

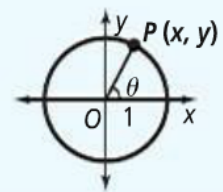
The **cosecant** (csc), **secant** (sec), and **cotangent** (cot) functions are defined using reciprocals. Their domains do not include the real numbers θ that make the denominator zero.

$$\text{csc } \theta = \frac{1}{\sin \theta} \qquad \text{sec } \theta = \frac{1}{\cos \theta} \qquad \text{cot } \theta = \frac{1}{\tan \theta}$$

(cot $\theta = 0$ at odd multiples of $\frac{\pi}{2}$, where tan θ is undefined.)

You can use the unit circle to evaluate the reciprocal trigonometric functions directly. Suppose the terminal side of an angle θ in standard position intersects the unit circle at the point (x, y) .

Then $\text{csc } \theta = \frac{1}{y}$, $\text{sec } \theta = \frac{1}{x}$, $\text{cot } \theta = \frac{x}{y}$.

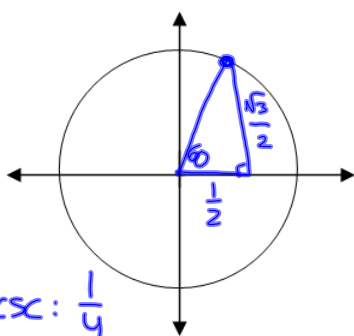


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Ex. 1 Find the exact value of each.

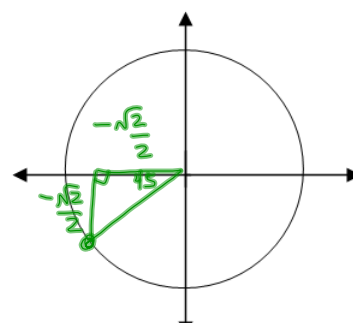
$\text{csc}\left(\frac{\pi}{3}\right)$ $\frac{1}{\frac{\sqrt{3}}{2}}$

$\text{sec}\left(\frac{5\pi}{4}\right)$ 225°



csc: $\frac{1}{\frac{\sqrt{3}}{2}}$

$\frac{1}{\frac{\sqrt{3}}{2}} \rightarrow \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$



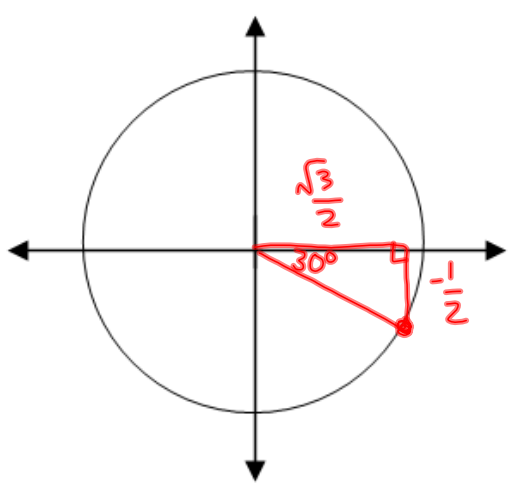
sec: $\frac{1}{x} = \frac{1}{-\frac{\sqrt{2}}{2}} = \frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$

rationalize

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$\cot\left(\frac{11\pi}{6}\right)$ 330°

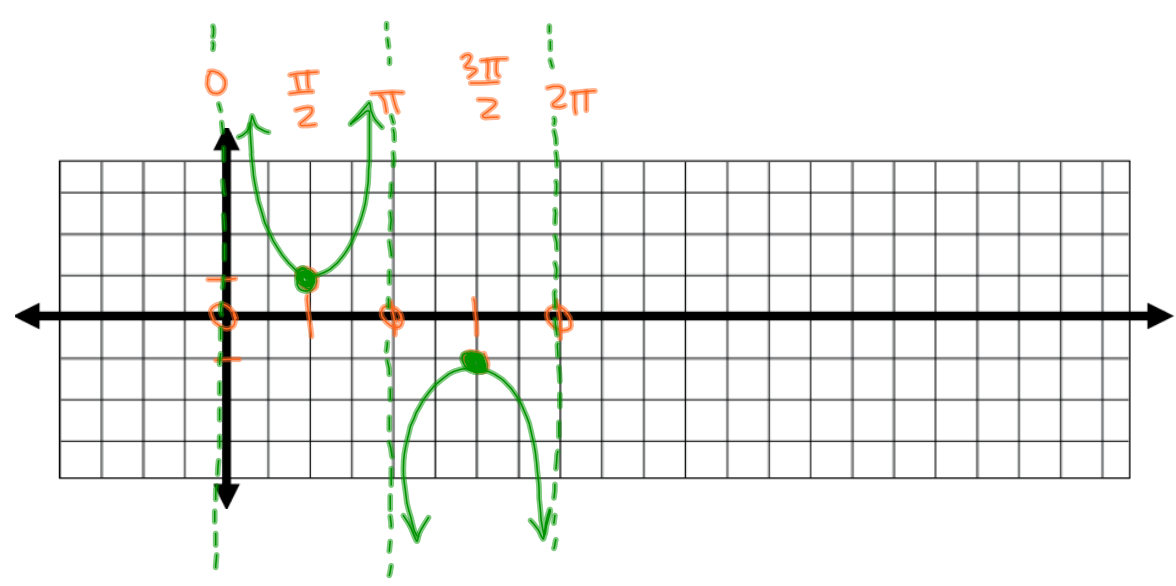
cot : $\frac{x}{y}$

$$= \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{-2}{1} = \boxed{-\sqrt{3}}$$


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Ex. 2 Graph from 0 to 2pi. Start with $y = \sin x$

$y = \csc x$

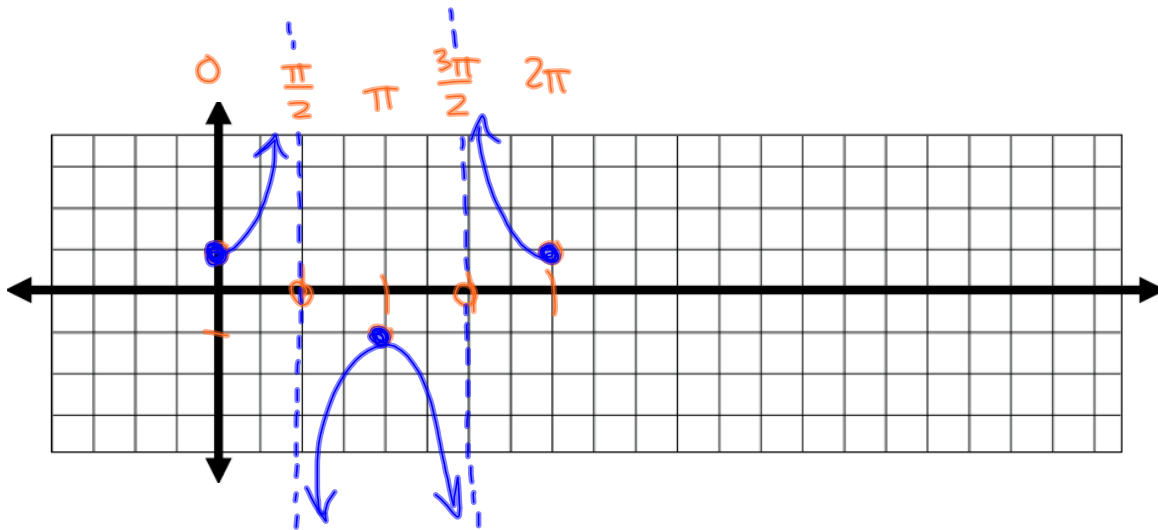


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Ex. 3 Graph from 0 to 2pi.

$y = \sec x$

$y = \cos x$

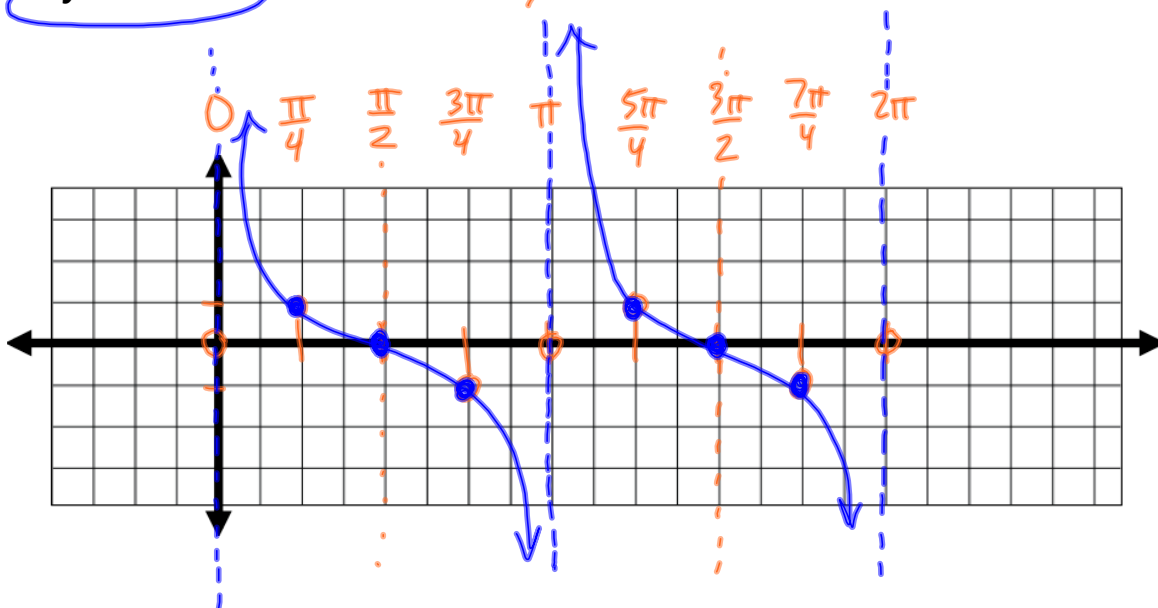


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Ex. 4 Graph from 0 to 2pi.

$y = \cot x$

$y = \tan x$



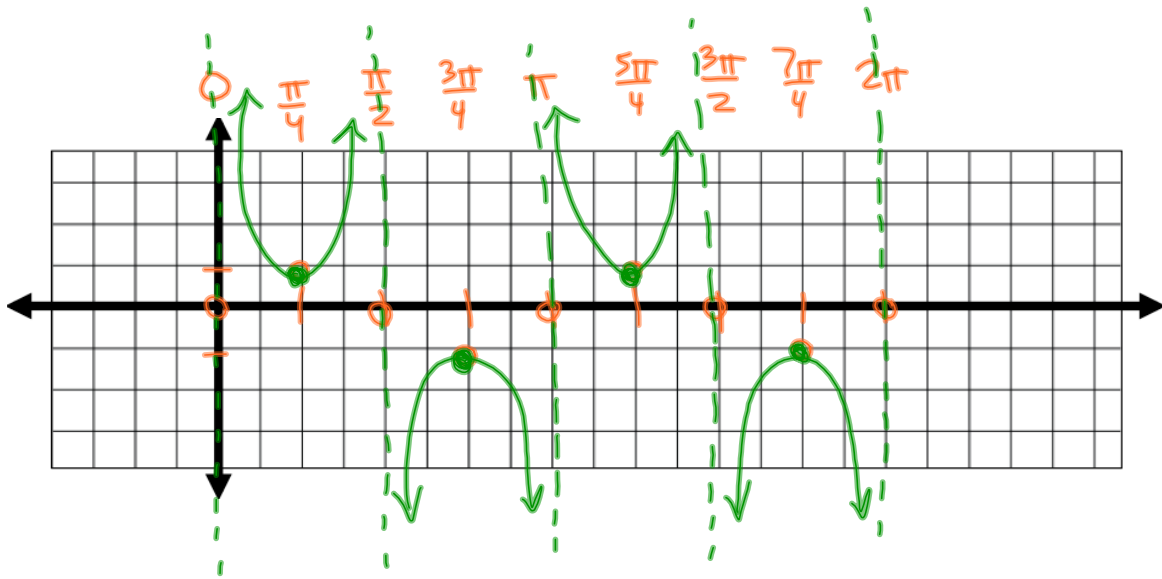
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Ex. 5 Graph from 0 to 2pi.

$y = \csc 2x$

$y = \sin 2\theta$

Period: π



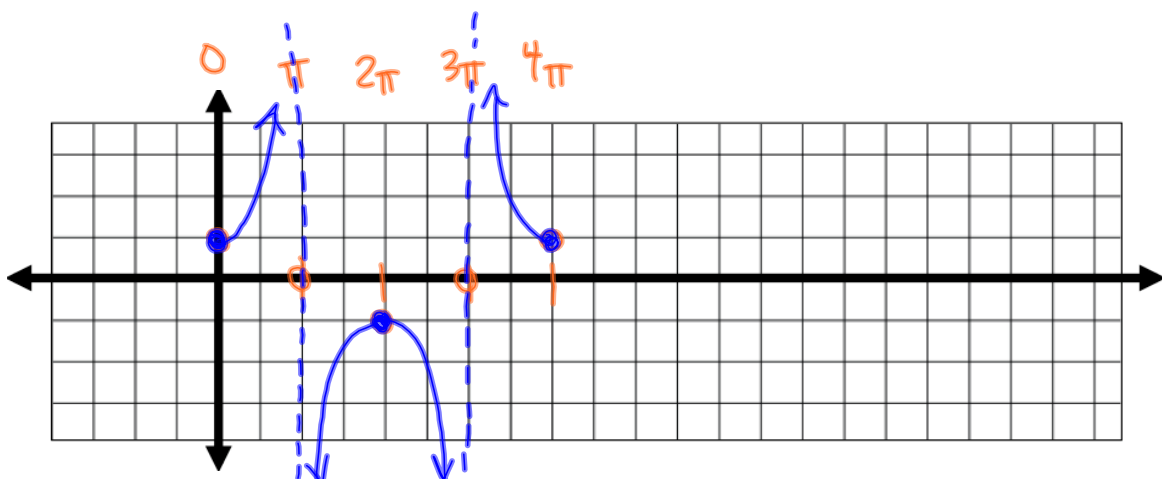
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Ex. 6 Graph from 0 to 2pi.

$y = \sec (1/2)x$

$y = \cos \frac{1}{2}x$

Period: 4π



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HW: p.888:

9 - 16 all, 25 - 28 all, 40 - 42 all, 44 - 46 all

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