Topic 2.1 - Graphs of Exponential and Logarithmic Functions

Day 1 - Graphs of Exponential Functions

Definition of the Exponential Function

The **exponential function** f **with base** b is defined by

$$f(x) = b^x \quad \text{or} \quad y = b^x,$$

where b is a positive constant other than 1 ($b > 0$ and $b \neq 1$) and x is any real number.

DID THIS ON FRIDAY!

- Here are some examples of exponential functions:

$f(x) = 2^x$ — Base is 2.

$g(x) = 10^x$ — Base is 10.

$h(x) = 3^{x+1}$ — Base is 3.

$j(x) = \left(\frac{1}{2}\right)^{x-1}$. — Base is $\frac{1}{2}$.

- By contrast, the following functions are not exponential functions:

$F(x) = x^2$ — Variable is the base and not the exponent.

$G(x) = 1^x$ — The base of an exponential function must be a positive constant other than 1.

$H(x) = (-1)^x$ — The base of an exponential function must be positive.

$J(x) = x^x$. — Variable is both the base and the exponent.

Graphing Exponential Functions

Ex1. Graph the function by creating a table of values. Then find the domain and range. Also, list the equation of the asymptote.

$f(x) = 2^x$

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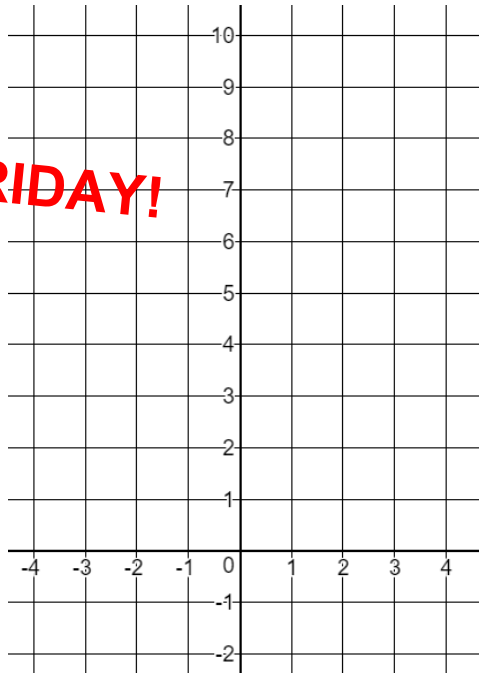

Ex2. Graph the function by creating a table of values. Then find the domain and range. Also, list the equation of the asymptote.

$$f(x)=\left(\frac{1}{2}\right)^x$$

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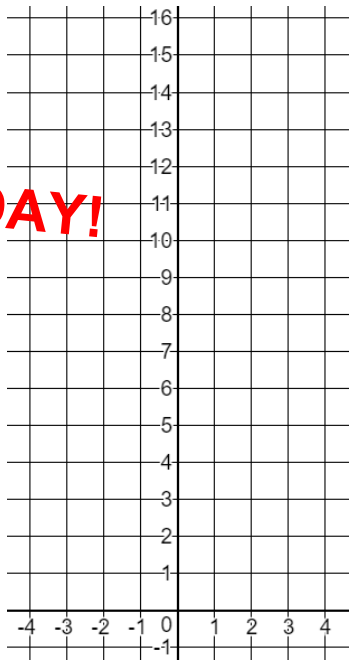


Transformations of Exponential Functions

The graphs of exponential functions can be translated vertically or horizontally, reflected, stretched, or shrunk. These transformations are summarized in **Table 3.1**.

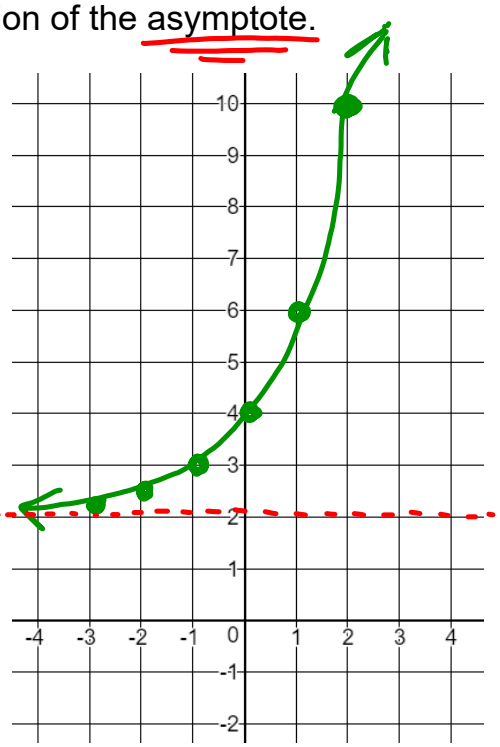
Table 3.1 Transformations Involving Exponential Functions

In each case, c represents a positive real number.

Transformation	Equation	Description
Vertical translation	$g(x) = b^x + c$ $g(x) = b^x - c$	• Shifts the graph of $f(x) = b^x$ upward c units. • Shifts the graph of $f(x) = b^x$ downward c units.
Horizontal translation	$g(x) = b^{x+c}$ $g(x) = b^{x-c}$	• Shifts the graph of $f(x) = b^x$ to the left c units. • Shifts the graph of $f(x) = b^x$ to the right c units.
Reflection	$g(x) = -b^x$ $g(x) = b^{-x}$	• Reflects the graph of $f(x) = b^x$ about the x-axis. • Reflects the graph of $f(x) = b^x$ about the y-axis.
Vertical stretching or shrinking	$g(x) = cb^x$	• Vertically stretches the graph of $f(x) = b^x$ if $c > 1$. • Vertically shrinks the graph of $f(x) = b^x$ if $0 < c < 1$.
Horizontal stretching or shrinking	$g(x) = b^{cx}$	• Horizontally shrinks the graph of $f(x) = b^x$ if $c > 1$. • Horizontally stretches the graph of $f(x) = b^x$ if $0 < c < 1$.

Ex3. Graph the function by creating a table of values. Then transform it. Then find the domain and range. Also, list the equation of the asymptote.

$$f(x) = 2^{x+1} + 2$$



Order of Transformations:

1. Start with parent exponential function points
2. Horizontal Shifting
3. Stretching or Shrinking
4. Reflecting
5. Vertical Shifting

Parent: $f(x) = 2^x$

Left 1, Up 2

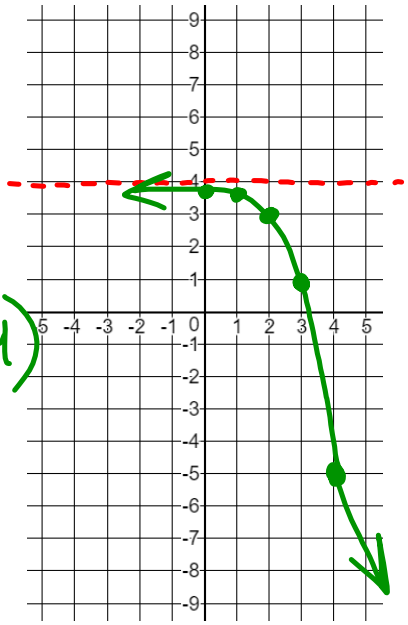
Asymptote: $y = 2$

D: $(-\infty, \infty)$

R: $(2, \infty)$

Ex4. Graph the function by creating a table of values. Then transform it. Then find the domain and range. Also, list the equation of the asymptote.

$$f(x) = -3^{x-2} + 4$$



Parent: $f(x) = 3^x$

Order of Transformations:

1. Start with parent exponential function points
2. Horizontal Shifting
3. Stretching or Shrinking
4. Reflecting
5. Vertical Shifting

Asym: $y = 4$

D: $(-\infty, \infty)$ R: $(-\infty, 4)$

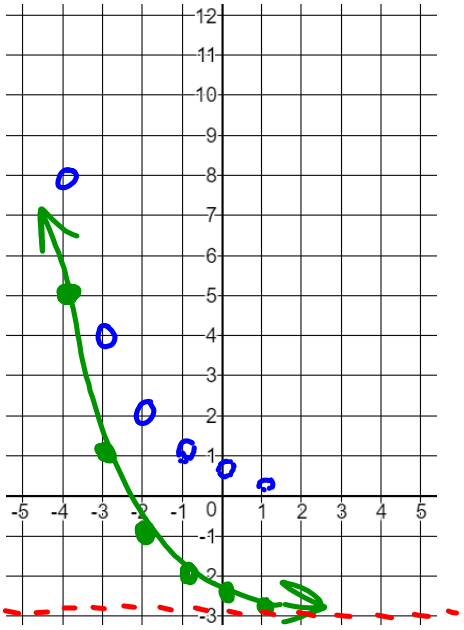
Right 2, reflect over x-axis, up 4

Ex5. Graph the function by creating a table of values. Then transform it. Then find the domain and range. Also, list the equation of the asymptote.

$$f(x)=\frac{1}{2}(2)^{-x}-3$$

Parent: $y=2^x$

1. Vertical shrink by $\frac{1}{2}$.
2. Reflect over y-axis.
3. Down 3.



Asym: $y=-3$

D: $(-\infty,\infty)$ R: $(-3,\infty)$