Topic 2.1 - Day 2 - Solving Exponential Equations

Before we get into solving, lets review powers of different numbers:

$y = 2^x$

x	y
5	32
4	16
3	8
2	4
1	2
0	1
-1	1/2
-2	1/4
-3	1/8

$y = 3^x$

x	y
5	243
4	81
3	27
2	9
1	3
0	1
-1	1/3
-2	1/9
-3	1/27

$y = 4^x$

x	y
5	1024
4	256
3	64
2	16
1	4
0	1
-1	1/4
-2	1/16
-3	1/64

$y = 5^x$

x	y
5	3125
4	625
3	125
2	25
1	5
0	1
-1	1/5
-2	1/25
-3	1/125

$y = 6^x$

x	y
5	7776
4	1296
3	216
2	36
1	6
0	1
-1	1/6
-2	1/36
-3	1/216

Exponential Equations

An **exponential equation** is an equation containing a variable in an exponent. Examples of exponential equations include

$$2^{3x-8} = 16, \quad 4^x = 15, \quad \text{and} \quad 40e^{0.6x} = 240.$$

Some exponential equations can be solved by expressing each side of the equation as a power of the same base. All exponential functions are one-to-one—that is, no two different ordered pairs have the same second component. Thus, if b is a positive number other than 1 and $b^M = b^N$, then $M = N$.

Solving Exponential Equations by Expressing Each Side as a Power of the Same Base

If $b^M = b^N$, then $M = N$.

Express each side as a power of the same base. | Set the exponents equal to each other.

1. Rewrite the equation in the form $b^M = b^N$.
2. Set $M = N$.
3. Solve for the variable.

Ex.1 Solve each exponential equation.

$2^{3x-8}=16$

$2^{3x-8}=2^4$

$3x-8=4$

$3x=12$

$\boxed{x=4}$

$5^{3x-6}=125$

$5^{3x-6}=5^3$

$3x-6=3$

$3x=9$

$\boxed{x=3}$

$27^{x+3}=9^{x-1}$

$3^{3(x+3)}=3^{2(x-1)}$

$3x+9=2x-2$

$-2x \quad -2x$

$x+9=-2$

$-9 \quad -9$

$\boxed{x=-11}$

$8^{x+2}=4^{x-3}$

$2^{3(x+2)}=2^{2(x-3)}$

$3x+6=2x-6$

$x+6=-6$

$\boxed{x=-12}$

Ex.2 Solve each exponential equation.

$\left(\frac{1}{64}\right)^{2a}=16$

$4^{-3(2a)}=4^2$

$-6a=2$

$-6 \quad -6$

$\boxed{a=-\frac{1}{3}}$

$\left(\frac{1}{6}\right)^{2x+2}=36$

$6^{-1(2x+2)}=6^2$

$-2x-2=2$

$-2x=4$

$\boxed{x=-2}$

$\left(\frac{1}{25}\right)^{-p}=\left(\frac{1}{5}\right)^{3-2p}$

$5^{-2(-p)}=5^{-1(3-2p)}$

$2p=-3+2p$

$-2p \quad -2p$

$0=-3$

$\boxed{\text{No Solution}}$