October 08, 2019

Graphs of Exponential and Logarithmic Functions

Topic 2.1 - Day 3 - Graphs of Logarithmic Functions

GREAT QUESTION!

You mentioned that the inverse of the exponential function is called the logarithmic function. We haven't discussed inverses of functions since Section 1.8. What should I already know about functions and their inverses?

Here's a brief summary:

1. Only one-to-one functions have inverses that are functions. A function, f, has an inverse function, f^{-1}, if there is no horizontal line that intersects the graph of f at more than one point.
2. If a function is one-to-one, its inverse function can be found by interchanging x and y in the function's equation and solving for y.
3. If $f(a) = b$, then $f^{-1}(b) = a$. The domain of f is the range of f^{-1}. The range of f is the domain of f^{-1}.
4. $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.
5. The graph of f^{-1} is the reflection of the graph of f about the line $y = x$.

The Definition of Logarithmic Functions

No horizontal line can be drawn that intersects the graph of an exponential function at more than one point. This means that the exponential function is one-to-one and has an inverse. Let's use our switch-and-solve strategy from Section 1.8 to find the inverse.

All exponential functions have inverse functions.

$f(x) = b^x$

Switch and Solve Strategy

Step 1 **Replace $f(x)$ with y:** $y = b^x$.

Step 2 **Interchange x and y:** $x = b^y$.

Step 3 **Solve for y:** ?

The question mark indicates that we do not have a method for solving $b^y = x$ for y. To isolate the exponent y, a new notation, called *logarithmic notation*, is needed. This notation gives us a way to name the inverse of $f(x) = b^x$. **The inverse function of the exponential function with base b is called the *logarithmic function with base b*.**

Definition of the Logarithmic Function

For $x > 0$ and $b > 0$, $b \neq 1$,

$$\boxed{y = \log_b x} \text{ is equivalent to } b^y = x.$$

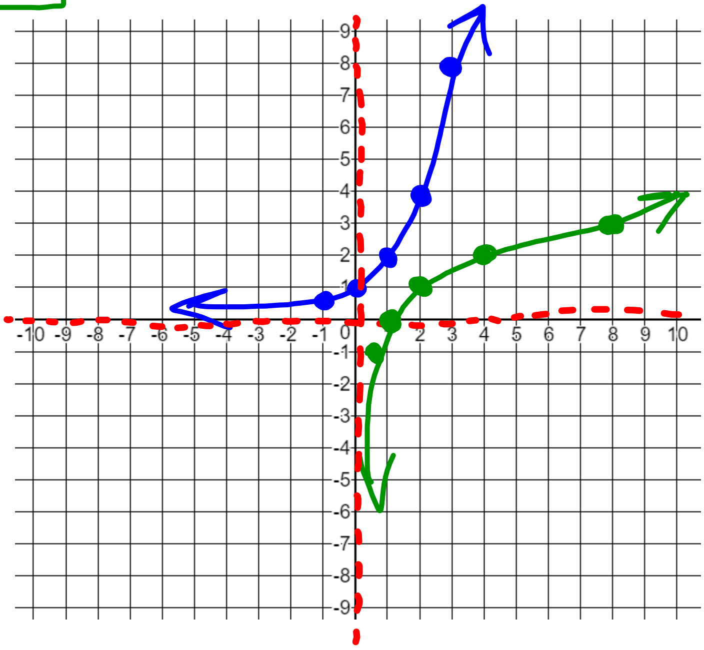
The function $f(x) = \log_b x$ is the **logarithmic function with base b**.

EXAMPLE Graphs of Exponential and Logarithmic Functions

Graph $\boxed{f(x) = 2^x}$ and $\boxed{g(x) = \log_2 x}$ in the same rectangular coordinate system.

D: $(-\infty, \infty)$
R: $(0, \infty)$
HA: $y = 0$

D: $(0, \infty)$
R: $(-\infty, \infty)$
VA: $x = 0$



Transformations of Log. Functions

Transformation	Equation	Description
Vertical translation	$g(x) = \log_b x + c$ $g(x) = \log_b x - c$	• Shifts the graph of $f(x) = \log_b x$ upward c units. • Shifts the graph of $f(x) = \log_b x$ downward c units.
Horizontal translation	$g(x) = \log_b(x + c)$ $g(x) = \log_b(x - c)$	• Shifts the graph of $f(x) = \log_b x$ to the left c units. Vertical asymptote: $x = -c$ • Shifts the graph of $f(x) = \log_b x$ to the right c units. Vertical asymptote: $x = c$
Reflection	$g(x) = -\log_b x$ $g(x) = \log_b(-x)$	• Reflects the graph of $f(x) = \log_b x$ about the x-axis. • Reflects the graph of $f(x) = \log_b x$ about the y-axis.
Vertical stretching or shrinking	$g(x) = c \log_b x$	• Vertically stretches the graph of $f(x) = \log_b x$ if $c > 1$. • Vertically shrinks the graph of $f(x) = \log_b x$ if $0 < c < 1$.
Horizontal stretching or shrinking	$g(x) = \log_b(cx)$	• Horizontally shrinks the graph of $f(x) = \log_b x$ if $c > 1$. • Horizontally stretches the graph of $f(x) = \log_b x$ if $0 < c < 1$.

Ex2. Graph the function by starting with the exponential parent function. Then invert it to create the logarithmic parent function. Then transform it. Then find the domain and range. Finally, list the equation of the asymptote.

$$f(x) = -\log_2(x+3)$$

Parent Expo : $y = 2^x$

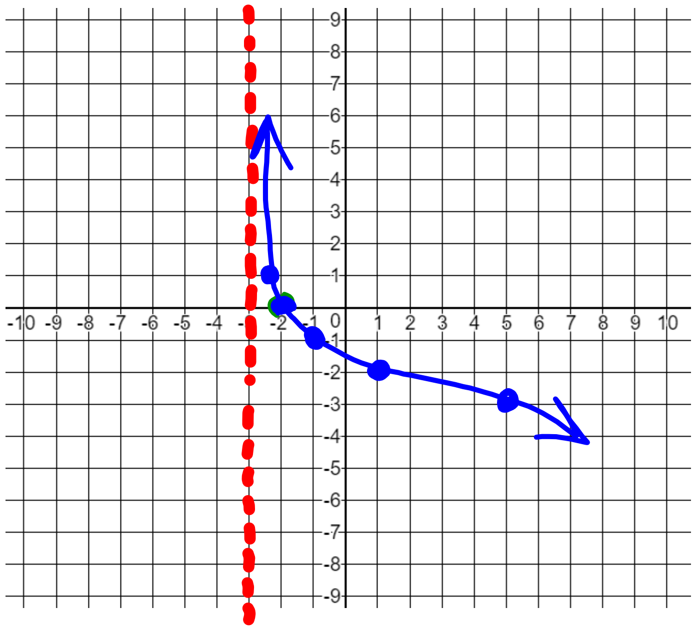
Transformations :

Left 3, reflect over x-axis.

D: $(-3, \infty)$

R: $(-\infty, \infty)$

VA : $x = -3$



Ex3. Graph the function by starting with the exponential parent function. Then invert it to create the logarithmic parent function. Then transform it. Then find the domain and range. Finally, list the equation of the asymptote.

$$f(x) = 3\log_{1/2} x + 4$$

Parent Expo : $y = \left(\frac{1}{2}\right)^x$

Transform : vertical stretch by 3, up 4.

D: $(0, \infty)$

R: $(-\infty, \infty)$

VA : $x = 0$