September 30, 2019

Topic 2.1 - Day 4 -

Special Bases and Evaluating Logs

Logarithmic Form: $\longrightarrow$ **Exponential Form:**

$$y = \log_b x \qquad\qquad x = b^y$$

base

exponent

EXAMPLE 1 Changing from Logarithmic to Exponential Form

Write each equation in its equivalent exponential form:

a. $2 = \log_5 x$ **b.** $3 = \log_b 64$ **c.** $\log_3 7 = y$.

$\boxed{5^2 = x}$ $\boxed{b^3 = 64}$ $\boxed{3^y = 7}$

Exponential Form: $\longrightarrow$ **Logarithmic Form:**

$$y = b^x \qquad\qquad x = \log_b y$$

exponent

base

Example 2: Changing from Exponential Form to logarithmic form.

Write each equation in its equivalent logarithmic form.

a. $5^3 = 125$ **b.** $49^{1/2} = 7$ **c.** $4^{-2} = 1/16$

$\boxed{\log_5 125 = 3}$ $\boxed{\log_{49} 7 = \frac{1}{2}}$ $\boxed{\log_4 \frac{1}{16} = -2}$

What is it? Natural Base "e"

The Natural Base e

An irrational number, symbolized by the letter e, appears as the base in many applied exponential functions. The number e is defined as the value that $\left(1+\frac{1}{n}\right)^n$ approaches as n gets larger and larger. **Table 3.2** shows values of $\left(1+\frac{1}{n}\right)^n$ for increasingly large values of n. As $n \to \infty$, the approximate value of e to nine decimal places is

$$e \approx 2.718281827.$$

The irrational number e, approximately 2.72, is called the **natural base**. The function $f(x) = e^x$ is called the **natural exponential function**.

Table 3.2

n	$\left(1+\frac{1}{n}\right)^n$
1	2
2	2.25
5	2.48832
10	2.59374246
100	2.704813829
1000	2.716923932
10,000	2.718145927
100,000	2.718268237
1,000,000	2.718280469
1,000,000,000	2.718281827

As $n \to \infty$, $\left(1+\frac{1}{n}\right)^n \to e$.

In calculus, this is expressed as $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$.

Natural Logarithms

Natural Logarithms

The logarithmic function with base e is called the **natural logarithmic function**. The function $f(x) = \log_e x$ is usually expressed as $f(x) = \ln x$, read "el en of x." A calculator with an $\boxed{\text{LN}}$ key can be used to evaluate natural logarithms.

Exponential Form ⟷ **Logarithmic Form**

$y = e^x$ $\qquad$ $x = \ln y$

ln has built in base e

Ex.3 Write in logarithmic form.

a. $y = e^3$ $\qquad$ $\boxed{\ln y = 3}$

b. $e^x = 55$ $\qquad$ $\boxed{\ln 55 = x}$

Ex.4 Write in exponential form.

a. $4 = \ln y$ $\qquad$ $e^4 = y$

b. $\ln 7 = x$ $\qquad$ $e^x = 7$

Common Log ("base 10")

Common Logarithms

The logarithmic function with base 10 is called the **common logarithmic function.** The function $f(x) = \log_{10} x$ is usually expressed as $f(x) = \log x$. A calculator with a LOG key can be used to evaluate common logarithms.

Exponential Form	Logarithmic Form
$y = 10^x$	$x = \log y$

log (with no base) has built in base *10*

Ex.5 Write in logarithmic form.

a. $y = 10^3$

$\boxed{\log y = 3}$

b. $10^x = 10000$

$\boxed{\log 10000 = x}$

Ex.6 Write in exponential form.

a. $2 = \log y$

$10^2 = y$

b. $\log 1000 = x$

$10^x = 1000$

Evaluating Logarithms

Example: Evaluate $\log_2 32$

"2 to what power equals 32 ?"

Can be written this way: $2^x = 32$

$2^x = 2^?$

Ex.7 Evaluate.

a.) $\log_5 625 = ?$

$5^x = 625$

$5^x = 5^4$

4

b.) $\log_{36} 6 = ?$

$36^x = 6$

$6^{2x} = 6^1$

$2x = 1$

$x = \frac{1}{2}$

$\frac{1}{2}$

c.) $\log_8 32 = ?$

$8^x = 32$

$2^{3x} = 2^5$

$3x = 5$

$\frac{5}{3}$

Ex.8 Evaluate these special logarithms.

a.) $\log\left(\dfrac{1}{100000}\right) = ?$

$10^x = \dfrac{1}{100000}$

$10^x = 10^{-5}$

-5

b.) $\ln e$

$\log_e e = x$

$e^x = e^1$

1

c.) $\log_2 = -32$

$2^x = -32$

no solution