

2.2 - Day 1 - Solving Logarithmic Equations

Logarithmic Equations

A **logarithmic equation** is an equation containing a variable in a logarithmic expression. Examples of logarithmic equations include

$$\log_4(x + 3) = 2 \quad \text{and} \quad \ln(x + 2) - \ln(4x + 3) = \ln\left(\frac{1}{x}\right).$$

Some logarithmic equations can be expressed in the form $\log_b M = c$. We can solve such equations by rewriting them in exponential form.

Ex1. Convert each into exponential form.

$$\log_3 243 = 5$$

$$3^5 = 243$$

$$\log_{10} 100 = 2$$

$$10^2 = 100$$

$$\ln M = 4$$

$$\log_e M = 4$$

$$e^4 = M$$

Using the Definition of a Logarithm to Solve Logarithmic Equations

- Express the equation in the form $\log_b M = c$.
- Use the definition of a logarithm to rewrite the equation in exponential form:

$$\log_b M = c \quad \text{means} \quad b^c = M.$$

Logarithms are exponents.

- Solve for the variable.
- Check proposed solutions in the original equation. Include in the solution set only values for which $M > 0$.

EXAMPLE 6 Solving Logarithmic Equations

Solve:

a. $\log_4(x + 3) = 2$

$$4^2 = x + 3$$

$$16 = x + 3$$

$$13 = x$$

b. $\frac{3 \cdot \ln(2x)}{3} = \frac{12}{3}$

$$\ln(2x) = 4$$

$$\log_e(2x) = 4$$

$$e^4 = \frac{2x}{2}$$

$$x = 27.2991$$

Logarithmic expressions are defined only for logarithms of positive real numbers. Always check proposed solutions of a logarithmic equation in the original equation. Exclude from the solution set any proposed solution that produces the logarithm of a negative number or the logarithm of 0. *** You also cannot have a logarithm with a negative or zero base!

Ex2. Solve.

$$-5 \log(n-7) - 6 = -16$$

$$\frac{-5 \cdot \log(n-7)}{-5} = \frac{-10}{-5}$$

$$\log_{10}(n-7) = 2$$

$$10^2 = n-7$$

$$100 = n-7$$

$$\boxed{107 = n}$$

$$\log_2(x^2 - 2x) = 3$$

$$2^3 = x^2 - 2x$$

$$8 = x^2 - 2x$$

$$0 = x^2 - 2x - 8 \quad \text{Factor.}$$

$$0 = (x-4)(x+2)$$

$$\boxed{x=4} \quad \boxed{x=-2}$$

Ex3. Solve.

~~$$\log_{13}(4n+9) = \log_{13}(2-3n)$$~~

~~$$4n+9 = 2-3n$$~~

~~$$\frac{+3n}{+3n} \quad \frac{+3n}{+3n}$$~~

~~$$7n+9 = 2$$~~

~~$$\frac{-9}{-9} \quad \frac{-9}{-9}$$~~

~~$$7n = -7$$~~

~~$$\boxed{n = -1}$$~~

~~$$\log_7(-10+2x^2) = \log_7(3x^2-7x)$$~~

~~$$-10+2x^2 = 3x^2-7x$$~~

~~$$\frac{-2x^2}{-2x^2} \quad \frac{-2x^2}{-2x^2}$$~~

~~$$-10 = x^2 - 7x$$~~

~~$$0 = x^2 - 7x + 10 \quad \text{Factor.}$$~~

~~$$0 = (x-2)(x-5)$$~~

~~$$\boxed{x=2} \quad \boxed{x=5}$$~~

