October 18, 2019

2.2 - Day 2 - Solving Logarithmic Equations w/ Props.

Properties for Condensing Logarithmic Expressions

For $M > 0$ and $N > 0$:

1. $\log_b M + \log_b N = \log_b (MN)$ Product rule
2. $\log_b M - \log_b N = \log_b \left(\frac{M}{N}\right)$ Quotient rule
3. $p \log_b M = \log_b M^p$ Power rule

Before we start:

Any questions from yesterday's HW?

EXAMPLE 7 Solving a Logarithmic Equation

Solve: $\log_2 x + \log_2(x - 7) = 3$

$\log_2 x^2 - 7x = 3$

$2^3 = x^2 - 7x$

$8 = x^2 - 7x$

$0 = x^2 - 7x - 8$

$0 = (x-8)(x+1)$

✓ $\boxed{x=8}$ $\boxed{\cancel{x=-1}}$

$\log_4(x+2) - \log_4(x-1) = 1$

$\log_4 \frac{x+2}{x-1} = 1$

$\frac{4^1}{1} = \frac{x+2}{x-1}$

$4x - 4 = x + 2$

$3x - 4 = 2$

$3x = 6$

$\boxed{x=2}$

Check Point 7 Solve:

$\log_2(x-3) - 3\log_2 2 = 4$

$\log_2(x-3) - \log_2 2^3 = 4$

$\log_2 \frac{x-3}{8} = 4$

$8 \cdot 2^4 = \frac{x-3}{8} \cdot 8$

$128 = x - 3$

$\boxed{131 = x}$

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Some logarithmic equations can be expressed in the form $\log_b M = \log_b N$, where the bases on both sides of the equation are the same. Because all logarithmic functions are one-to-one, we can conclude that $M = N$.

Using the One-to-One Property of Logarithms to Solve Logarithmic Equations

1. Express the equation in the form $\log_b M = \log_b N$. This form involves a single logarithm whose coefficient is 1 on each side of the equation.
2. Use the one-to-one property to rewrite the equation without logarithms: If $\log_b M = \log_b N$, then $M = N$.
3. Solve for the variable.
4. Check proposed solutions in the original equation. Include in the solution set only values for which $M > 0$ and $N > 0$.

EXAMPLE 8 Solve.

$$\log(5x+1) = \log(2x+3) + \log 2$$

Product Rule

$$\log(5x+1) = \log 4x+6$$

$$5x+1 = 4x+6$$
$$-4x \quad -4x$$

$$x+1 = 6$$
$$-1 \quad -1$$

$$\boxed{x=5}$$

Check Point 8 Solve: $\ln(x+2) - \ln(4x+3) = \ln\left(\frac{1}{x}\right)$

$$\ln \frac{x+2}{4x+3} = \ln\left(\frac{1}{x}\right)$$

$$\frac{(x+2)}{4x+3} = \frac{1}{x}$$

$$x^2+2x = 4x+3$$
$$-4x \quad -4x \; -3$$

$$x^2-2x-3=0$$

$$(x-3)(x+1)=0$$

$$\boxed{x=3} \quad \cancel{\boxed{x=-1}}$$

✓