2.2 Day 3 - Solving Exponential Equations

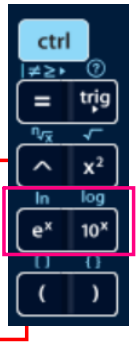
Most exponential equations cannot be rewritten so that each side has the same base. Here are two examples:

$4^x = 15$

We cannot rewrite both sides in terms of base 2 or base 4.

$10^x = 120{,}000.$

We cannot rewrite both sides in terms of base 10.



The Change-of-Base Property: Introducing Common and Natural Logarithms

Introducing Common Logarithms	Introducing Natural Logarithms
$\log_b M = \frac{\log M}{\log b}$	$\log_b M = \frac{\ln M}{\ln b}$

Ex. Solve:

$10^x = 120000$

log form

$\log_{10}(120000) = x$

$\frac{\ln(120000)}{\ln(10)} = x$

$x = 5.0792$

$4^x = 15$

log form

$\log_4(15) = x$

$\frac{\ln(15)}{\ln(4)} = x$

$x = 1.9534$

Check Point 1 Solve:

1. Isolate the exponential expression.
2. Convert the equation into logarithmic form.
3. Solve for the variable using change of base.

$5^{x-3} + 4 = 138$

$-4 \quad -4$

$5^{x-3} = 134$

$\log_5(134) = x - 3$

$\frac{\ln(134)}{\ln(5)} = x - 3$

$+3 \quad +3$

$x = 6.0432$

$\frac{3 \cdot 10^{2x}}{3} = \frac{42}{3}$

$10^{2x} = 14$

log form

$\log_{10}(14) = 2x$

$\frac{\ln(14)}{\ln(10)} = 2x$

$\div 2 \quad \div 2$

$x = 0.5731$

Ex. Solve:

1. Isolate the exponential expression.
2. Convert the equation into logarithmic form.
3. Solve for the variable using change of base.

$$40e^{0.5x} - 3 = 237$$

$+3 \quad +3$

$$\frac{40e^{0.5x}}{40} = \frac{240}{40}$$

$$e^{0.5x} = 6$$

log form

$$\log_e(6) = 0.5x$$

$$\frac{\ln(6)}{0.5} = \frac{0.5x}{0.5}$$

$$x = 3.5835$$