Topic 2.3 Day1 - Log/Expo Apps and Modeling

Compound Interest

Everyone needs a calculator for this unit!

We all want a wonderful life with fulfilling work, good health, and loving relationships. And let's be honest: Financial security wouldn't hurt! Achieving this goal depends on understanding how money in savings accounts grows in remarkable ways as a result of *compound interest*. **Compound interest** is interest computed on your original investment as well as on any accumulated interest.

Formulas for Compound Interest

After t years, the balance, A, in an account with principal P and annual interest rate r (in decimal form) is given by the following formulas:

1. For n compounding periods per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
2. For continuous compounding: $A = Pe^{rt}$.

Example 1: A sum of \$10,000 is invested at an annual rate of 8%. Find the balance in the account after 5 years subject to **a.** monthly compounding and **b.** continuous compounding.

a.) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 10000\left(1 + \left(\frac{.08}{12}\right)\right)^{(12 \cdot 5)}$

$A = \$14898.46$

b.) $A = P \cdot e^{rt}$

$A = 10000 \cdot e^{(.08 \cdot 5)}$

$A = \$14918.25$

Example 2: How long, to the nearest tenth of a year, will it take \$1000 to grow to \$3600 at 5.6% annual interest compounded quarterly?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$\frac{3600}{1000} = \frac{1000}{1000}\left(1+\left(\frac{.056}{4}\right)\right)^{4t}$

$3.6 = (1.014)^{4t}$

$\log_{1.014}(3.6) = 4t$

$\frac{\ln(3.6)}{\ln(1.014)} = 4t$

$\frac{\ln(3.6)}{\ln(1.014)} \Big/ 4 = 4t \Big/ 4$

$t = 23.0$ yrs.

Example 3: The formula $A = 37.3e^{0.0095t}$ models the population of California, A, in millions, t years after 2010.

a. What was the population of California in 2010?

b. When will the population of California reach 50 million?

a.) 37.3 million

b.) $\frac{50}{37.3} = \frac{37.3 \cdot e^{0.0095t}}{37.3}$

$\frac{50}{37.3} = e^{0.0095t}$

$\log_e\left(\frac{50}{37.3}\right) = 0.0095t$

$\frac{\ln\left(\frac{50}{37.3}\right)}{0.0095} = \frac{0.0095t}{0.0095}$

$t = 30.845\ldots$

$t = 31$ yrs.

2010 + 31 yrs : Yr. 2041

Answers to Day 1 HW 2.3

1.) A = $ 7761.13

2.) A = $ 1562.39

3.) A = $ 11815.59

4.) A = $ 27331.78

5.) P = $ 6547.50

6.) Yr. 2075

7.) t = 14.5 yrs.

8.) t = 13.9 yrs.

9.) t = 15.9 yrs.

10.) t = 14.6 yrs.

11.) rate = 9.8%

12.) rate = 40.2%