

Topic 2.3 Day1 - Log/Expo Apps and Modeling

Compound Interest

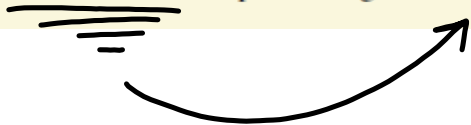
Everyone needs a calculator for this unit!

We all want a wonderful life with fulfilling work, good health, and loving relationships. And let's be honest: Financial security wouldn't hurt! Achieving this goal depends on understanding how money in savings accounts grows in remarkable ways as a result of *compound interest*. **Compound interest** is interest computed on your original investment as well as on any accumulated interest.

Formulas for Compound Interest

After t years, the balance, A , in an account with principal P and annual interest rate r (in decimal form) is given by the following formulas:

1. For n compounding periods per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
2. For continuous compounding: $A = Pe^{rt}$.



Example 1: A sum of \$10,000 is invested at an annual rate of 8%. Find the balance in the account after 5 years subject to a. monthly compounding and b. continuous compounding.

$$a.) A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 10000\left(1 + \left(\frac{.08}{12}\right)\right)^{(12 \cdot 5)}$$

$$A = \$14898.46$$

$$b.) A = P \cdot e^{rt}$$

$$A = 10000 \cdot e^{(.08 \cdot 5)}$$

$$A = \$14918.25$$

Example 2: How long, to the nearest tenth of a year, will it take \$1000 to grow to \$3600 at 5.6% annual interest compounded quarterly?

$$\frac{3600}{1000} = \frac{1000}{1000} \left(1 + \left(\frac{.056}{4} \right)^{4t} \right)$$

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$3.6 = (1.014)^{4t}$$

$$\log_{1.014}(3.6) = 4t$$

$$\frac{\ln(3.6)}{\ln(1.014)} = \frac{4t}{4}$$

$$t = 23.0 \text{ yrs.}$$

Example 3: The formula $A = 37.3e^{0.0095t}$ models the population of California, A , in millions, t years after 2010.

- What was the population of California in 2010?
- When will the population of California reach 50 million? 2010 + 31 yrs : Yr. 2041

a.) 37.3 million

b.) $\frac{50}{37.3} = \frac{37.3}{37.3} \cdot e^{0.0095t}$

$$\frac{50}{37.3} = e^{0.0095t}$$

$$\log_e \left(\frac{50}{37.3} \right) = 0.0095t$$

$$\frac{\ln \left(\frac{50}{37.3} \right)}{0.0095} = \frac{0.0095t}{0.0095}$$

$$t = 30.845 \dots$$

$$t = 31 \text{ yrs.}$$

Answers to Day 1 HW 2.3

- | | |
|-----------------------|-----------------------|
| 1.) $A = \$ 7761.13$ | 2.) $A = \$ 1562.39$ |
| 3.) $A = \$ 11815.59$ | 4.) $A = \$ 27331.78$ |
| 5.) $P = \$ 6547.50$ | 6.) Yr. 2075 |
| 7.) $t = 14.5$ yrs. | 8.) $t = 13.9$ yrs. |
| 9.) $t = 15.9$ yrs. | 10.) $t = 14.6$ yrs. |
| 11.) rate = 9.8% | 12.) rate = 40.2% |