October 17, 2019

# Topic 2.3 Day1 - Log/Expo Apps and Modeling

## Compound Interest

*Everyone needs a calculator for this unit!*

We all want a wonderful life with fulfilling work, good health, and loving relationships. And let's be honest: Financial security wouldn't hurt! Achieving this goal depends on understanding how money in savings accounts grows in remarkable ways as a result of *compound interest*. **Compound interest** is interest computed on your original investment as well as on any accumulated interest.

### Formulas for Compound Interest

After $t$ years, the balance, $A$, in an account with principal $P$ and annual interest rate $r$ (in decimal form) is given by the following formulas:

1. For $n$ compounding periods per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
2. For continuous compounding: $A = Pe^{rt}$.

**Example 1:** A sum of \$10,000 is invested at an annual rate of 8%. Find the balance in the account after 5 years subject to **a.** monthly compounding and **b.** continuous compounding.

a.) $A = 10000\left(1 + \left(\frac{.08}{12}\right)\right)^{(12 \cdot 5)}$

$A = \$14,898.46$

b.) $A = 10000 \cdot e^{(.08 \cdot 5)}$

$\$14,918.25$

**Example 2:** How long, to the nearest tenth of a year, will it take \$1000 to grow to \$3600 at 5.6% annual interest compounded quarterly?

$$\frac{3600}{1000} = \frac{1000}{1000}\left(1 + \frac{.056}{4}\right)^{4t}$$

$$3.6 = (1.014)^{4t}$$

log form: $\log_{1.014}(3.6) = 4t$

$$\frac{\ln(3.6)}{\ln(1.014)} = 4t$$

$$\frac{\ln(3.6)}{\ln(1.014)} \Big/ 4 = 4t \Big/ 4$$

$$t = 23.03\ldots$$

$t = 23$ yrs.

**Example 3:** The formula $A = 37.3e^{0.0095t}$ models the population of California, $A$, in millions, $t$ years after 2010.

**a.** What was the population of California in 2010?

**b.** When will the population of California reach 50 million?

a.) 37.3 million

b.) $\frac{50}{37.3} = \frac{37.3 \cdot e^{0.0095t}}{37.3}$

$$\frac{50}{37.3} = e^{0.0095t}$$

ln form: $\log_e$

$$\frac{\ln\left(\frac{50}{37.3}\right)}{0.0095} = \frac{0.0095t\ldots}{0.0095}$$

$t = 30.85\ldots$

$t = 31$ yrs.

Yr. 2041