2.3 Day2 - Log/Expo Apps and Modeling

Over a period of time, a cup of hot coffee cools to the temperature of the surrounding air. **Newton's Law of Cooling**, named after Sir Isaac Newton, states that the temperature of a heated object decreases exponentially over time toward the temperature of the surrounding medium.

NEWTON'S LAW OF COOLING

The function $T = C + (T_0 - C)e^{kt}$ models Newton's Law of Cooling. It allows you to predict the temperature T of an object, t time after it is placed in a constant-temperature cooling environment.

T_0 is the initial temperature of the object, and C is the constant temperature inside the room where the object is. The number k is a cooling constant for the particular object in question.

T = temperature of an object after time has passed

C = constant temperature of the room object is in

T_0 = temperature of the object initially

k = cooling constant

t = time that has passed

EXAMPLE Using Newton's Law of Cooling $T = C + (T_0 - C)e^{kt}$

A cake removed from the oven has a temperature of 210°F. It is left to cool in a room that has a temperature of 70°F. After 30 minutes, the temperature of the cake is 140°F.

a. Use Newton's Law of Cooling to find a model for the temperature of the cake, T, after t minutes. $\boxed{K = -0.0231}$

c. When will the temperature of the cake be 90°F?

a.) Find K:

$140 = 70 + (210 - 70) \cdot e^{30K}$

$-70 \quad -70$

$70 = 140 \cdot e^{30K}$

$\frac{70}{140} = \frac{140 \cdot e^{30K}}{140}$

$\frac{1}{2} = e^{30K}$

$\frac{\ln\left(\frac{1}{2}\right)}{30} = \frac{30K}{30}$

c.)

$90 = 70 + (210 - 70) \cdot e^{Kt}$

$20 = 140 \cdot e^{Kt}$

$\frac{20}{140} = e^{Kt}$

$\frac{\ln\left(\frac{20}{140}\right)}{-0.0231} = \frac{-0.0231t}{-0.0231}$

$\boxed{t = 84.2 \text{ min}}$

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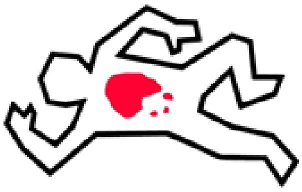
A detective is called to the scene of a crime where a dead body has just been found. She arrives on the scene at 10:23 pm and begins her investigation. Immediately, the temperature of the body is taken and is found to be 80° F. The detective checks the programmable thermostat and finds that the room has been kept at a constant 70° F for the past 3 days.



$$T = C + (T_0 - C)e^{kt}$$

After evidence from the crime scene is collected, the temperature of the body is taken once more and found to be 78.5° F. This last temperature reading was taken exactly one hour after the first one. The next day the detective is asked by another investigator, **"What time did our victim die?"** Assuming that the victim's body temperature was normal (98.6° F) prior to death, what is her answer to this question? Newton's Law of Cooling can be used to determine a victim's time of death.

PART I: Find *k* $K = -0.0027$

PART II: Find *t*, the time of death:

$$T = C + (T_0 - C)e^{kt}$$

$$78.5 = 70 + (80-70)\cdot e^{60k}$$

$$8.5 = 10\cdot e^{60k}$$

$$\frac{8.5}{10} = e^{60k}$$

$$\frac{\ln\left(\frac{8.5}{10}\right)}{60} = \frac{60k}{60}$$

$$80 = 70 + (98.6-70)\cdot e^{kt}$$

$$10 = 28.6\cdot e^{kt}$$

$$\frac{10}{28.6} = e^{kt}$$

$$\frac{\ln\left(\frac{10}{28.6}\right)}{-0.0027} = \frac{-0.0027t}{-0.0027}$$

$$\boxed{t = 389 \text{ min.}}$$

6 hrs 29 min

10:23 PM $\xrightarrow{-6\text{hrs.}}$ 4:23 PM

− 29 min

$\boxed{\text{T.O.D.: 3:54 PM}}$