2.3 Day3 - Other Log/Expo Apps and Modeling

Basic Exponential Growth or Decay Model:

$$A = A_o(1 \pm r)^t$$

A = the amount after t time A_o = the initial amount

t = time gone by r = the rate of growth (+) or decay (-)

Ex.1) Write an exponential model for this situation and solve.

In Titusville, FL there were 43,581 people in 2005. Since 2005, the population has been steadily decreasing at a rate of about 1.1% per year. If this trend continued, what will be the expected population in 2030? *(Round to the nearest person)*

$A = ?$

$A_o = 43581$

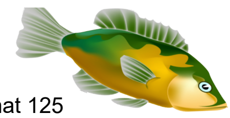
$r = .011$

$t = 25$ yrs.

$A = 43581(1 - .011)^{25}$

$\boxed{A = 33,053 \text{ people}}$

Ex.2) Write an exponential model for these situations and solve.



a.) A certain population of invasive foreign fish in a lake has been steadily increasing at a rate of 13% each year since 2006. Initially it is estimated that 125 of these fish were introduced to the lake. In what year will it be for this population to reach 10,000? *(Round to the nearest year)*

$r = .13$

$A_o = 125$

$A = 10000$

$t = ?$

$\frac{10000}{125} = \frac{125}{125}(1 + .13)^t$

$80 = (1.13)^t$

$\log_{1.13}(80) = t$

$\frac{\ln(80)}{\ln(1.13)} = t$

$t = 35.85...$

$t = 36$ yrs.

$\boxed{\text{Yr. } 2042}$

b.) Crazy Rabbits! The number of rabbits in Elkgrove has been doubling every month. There were 20 rabbits present initially. How long will it take for the population to become 500,000 ?

$A_o = 20$

$A = 500,000$

$r = 100\% \rightarrow 1$

$t = ?$

$500000 = 20(1 + 1)^t$

$25000 = (2)^t$

$\log_2(25000) = t$

$\frac{\ln(25000)}{\ln(2)} = t$

$\boxed{t = 14.6 \text{ months}}$

Half-Life Model:

$$A = A_o(0.5)^{\frac{t}{k}}$$

A = the amount after t time $\quad$ A_o = the initial amount

t = time gone by $\quad$ k = the half-life time

Ex. 3) Write an exponential model for this situation and solve.

The half-life of a certain radioactive substance is 14 days. The are 6.6 grams present initially. How long exactly until there will be 1 gram remaining?

$k = 14$ days

$A_o = 6.6$ g

$t = ?$

$A = 1$ g

$$\frac{1}{6.6} = \frac{6.6(0.5)^{\frac{t}{14}}}{6.6}$$

$$\frac{1}{6.6} = (0.5)^{\frac{t}{14}}$$

$$\log_{0.5}\left(\frac{1}{6.6}\right) = \frac{t}{14}$$

$$14 \times \frac{\ln\left(\frac{1}{6.6}\right)}{\ln(0.5)} = \frac{t}{14} \cdot 14$$

$$t = 38.11452434\ldots$$

$\boxed{\text{38 days, 2 hrs, 44 min, 55s.}}$

Ex. 4) Write an exponential model for the situations and solve.

The half-life of a certain radioactive substance is 2.5 seconds. $k =$

a.) If an initial amount of 1 kilogram is present, how long until that amount is 2 grams? *(Round to the nearest tenth of a second)*

b.) Determine the initial amount if there was 1 gram left after 1 minute.

$k = 2.5$ s.

$A_o = ?$

$A = 1$ g

$t = 1$ min $\rightsquigarrow$ 60s.

$$1 = A_o(0.5)^{60/2.5}$$

$$\frac{1}{(0.5)^{24}} = \frac{A_o(0.5)^{24}}{(0.5)^{24}}$$

$\boxed{A_o = 16{,}777{,}216 \text{ grams}}$