2.3 Day2 - Log/Expo Apps and Modeling

Over a period of time, a cup of hot coffee cools to the temperature of the surrounding air. **Newton's Law of Cooling**, named after Sir Isaac Newton, states that the temperature of a heated object decreases exponentially over time toward the temperature of the surrounding medium.

NEWTON'S LAW OF COOLING

The function $T = C + (T_0 - C)e^{kt}$ models Newton's Law of Cooling. It allows you to predict the temperature T of an object, t time after it is placed in a constant-temperature cooling environment.

T_0 is the initial temperature of the object, and C is the constant temperature inside the room where the object is. The number k is a cooling constant for the particular object in question.

T = temperature of an object after time has passed

C = constant temperature of the room object is in

T_0 = temperature of the object initially

k = cooling constant

t = time that has passed

EXAMPLE Using Newton's Law of Cooling $T = C + (T_0 - C)e^{kt}$

A cake removed from the oven has a temperature of 210°F. It is left to cool in a room that has a temperature of 70°F. After 30 minutes, the temperature of the cake is 140°F.

a. Use Newton's Law of Cooling to find a model for the temperature of the cake, T, after t minutes.

c. When will the temperature of the cake be 90°F?

a.) $140 = 70 + (210 - 70) \cdot e^{30k}$

$-70 \quad -70$

$70 = 140 \cdot e^{30k}$

$\frac{70}{140} = \frac{140 \cdot e^{30k}}{140}$

$0.5 = e^{30k}$

$\log_e(0.5) = 30k$

$\frac{\ln(0.5)}{30} = \frac{30k}{30}$

$\boxed{k = -0.0231}$

c.) $90 = 70 + (210 - 70) \cdot e^{kt}$

$20 = 140 \cdot e^{kt}$

$\frac{1}{7} = e^{kt}$

$\frac{\ln\left(\frac{1}{7}\right)}{-0.0231} = \frac{-0.0231t}{-0.0231}$

$\boxed{t = 84.2 \text{ min.}}$

Check Point

$$T = C + (T_0 - C)e^{kt}$$

A frozen steak initially has a temperature of 24°F. It is left to thaw in a room that has a temperature of 65°F. After 10 minutes, the temperature of the steak has risen to 30°F. After how many minutes will the temperature of the steak be 45°F?

a. Find, *k*, the constant.

b. Find, *t*, the time it will take the steak to be 45°F.

a.) $30 = 65 + (24-65)\cdot e^{10k}$

$-65 \quad -65$

$\frac{-35}{-41} = \frac{-41 \cdot e^{10k}}{-41}$

$\frac{35}{41} = e^{10k}$

$\frac{\ln\left(\frac{35}{41}\right)}{10} = \frac{10k}{10}$

$\boxed{k = -0.0158}$

b.) $45 = 65 + (24-65)\cdot e^{kt}$

$-20 = -41 \cdot e^{kt}$

$\frac{20}{41} = e^{kt}$

$\frac{\ln\left(\frac{20}{41}\right)}{-0.0158} = \frac{-0.0158t}{-0.0158}$

$\boxed{t = 45.4 \text{ min}}$

HW 2.3 - Day 2 Answers

3.)a.) k = -0.0419

b.) t = 69.6 minutes

4.)a.) k = -0.1643

b.) t = 11.5 minutes