

## Topic 3.1 - Day 3 - Geometric Sequences

HW Day 3: p. 1036:

1 - 23 odds, 63, 64, 65, 67

### Definition of a Geometric Sequence

A **geometric sequence** is a sequence in which each term after the first is obtained by multiplying the preceding term by a fixed nonzero constant. The amount by which we multiply each time is called the **common ratio** of the sequence.

The common ratio,  $r$ , is found by dividing any term after the first term by the term that directly precedes it. In the following examples, the common ratio is found by dividing the second term by the first term,  $\frac{a_2}{a_1}$ .

#### Geometric sequence

1, 5, 25, 125, 625, ...

4, 8, 16, 32, 64, ...

6, -12, 24, -48, 96, ...

9, -3, 1,  $-\frac{1}{3}$ ,  $\frac{1}{9}$ , ...

#### Common ratio

$$r = \frac{5}{1} = 5$$

$$r = \frac{8}{4} = 2$$

$$r = \frac{-12}{6} = -2$$

$$r = \frac{-3}{9} = -\frac{1}{3}$$

#### Recursive Formula:

$$a_n = a_{n-1} \cdot r$$

$r$  = common ratio

"Multiply  $r$  to the term in any position to get the next term."

### EXAMPLE 1 Writing the Terms of a Geometric Sequence

Write the first six terms of the geometric sequence with first term 6 and common ratio  $\frac{1}{3}$ .

$$6, 2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \frac{2}{81}, \dots$$

$$6\left(\frac{1}{3}\right) \quad 2\left(\frac{1}{3}\right) \quad \frac{2}{3}\left(\frac{1}{3}\right) \quad \frac{2}{9}\left(\frac{1}{3}\right) \quad \frac{2}{27}\left(\frac{1}{3}\right)$$

✓ **Check Point 1** Write the first six terms of the geometric sequence with first term 12 and common ratio  $\frac{1}{2}$ .

$$12, 6, 3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \dots$$

$$3\left(\frac{1}{2}\right) \quad \frac{3}{2}\left(\frac{1}{2}\right) \quad \frac{3}{4}\left(\frac{1}{2}\right)$$

### General Term of a Geometric Sequence

The  $n$ th term (the general term) of a geometric sequence with first term  $a_1$  and common ratio  $r$  is

**Explicit Formula:**

$$a_n = a_1 r^{n-1}$$

**EXAMPLE 2** Using the Formula for the General Term of a Geometric Sequence

$$n=8$$

$$a_1 = -4$$

Find the eighth term of the geometric sequence whose first term is  $-4$  and whose common ratio is  $-2$ .

$$r = -2 \quad a_8 = -4(-2)^7 = \boxed{512}$$

**Check Point 2** Find the seventh term of the geometric sequence whose first term is  $5$  and whose common ratio is  $-3$ .

$$a_7 = 5(-3)^6 = \boxed{3645}$$

$\begin{matrix} \uparrow & \uparrow \\ a_1 & r \end{matrix}$

**Check Point 3** Write the general term for the geometric sequence

$$a_1 = 3 \longrightarrow 3, 6, 12, 24, 48, \dots \quad r = 2$$

Then use the formula for the general term to find the eighth term.

$$a_n = 3(2)^{n-1} \quad a_8 = 3(2)^7 \quad \begin{matrix} \uparrow \\ n=8 \end{matrix}$$

$$\boxed{a_8 = 384}$$

**Example 4:** Find  $a_2$  and  $a_3$  for the geometric sequence.

$$3, a_2, a_3, -192, \dots$$

$$a_n = a_1(r)^{n-1}$$

$$a_4 = -192 \quad n=4$$

$$\frac{-192}{3} = \frac{3(r)^{4-1}}{3}$$

$$\sqrt[3]{-64} = \sqrt[3]{r^3}$$

$$r = -4$$

$$\boxed{\begin{matrix} a_2 = -12 \\ a_3 = 48 \end{matrix}}$$

**Answers to HW Day 3: #'s 1 - 23  
odds, 63, 64, 65, 67**

1.) 5, 15, 45, 135, 405, 1215

3.) 20, 10, 5,  $5/2$ ,  $5/4$ ,  $5/8$

5.) 10, -40, 160, -640, 2560, -10240

7.) -6, 30, -150, 750, -3750, 18750

9.) 768

11.) -10240

13.) 0.000000002

15.) 0.1

17.)  $a_n = 3(4)^{n-1}$ ; 12288

19.)  $a_n = 18(1/3)^{n-1}$ ;  $2/81$

21.)  $a_n = 1.5(-2)^{n-1}$ ; 96

23.)  $a_n = 0.0004(-10)^{n-1}$ ; 400

63.)  $a_2 = 12$ ,  $a_3 = 18$

64.)  $a_2 = -6$ ,  $a_3 = 18$

65.) \$ 16,384

67.) \$ 3,795,957.06

*Test Wednesday!*

