

3.2: - Summations and Series - Day 1

The Sum of the First n Terms of an Arithmetic Sequence

The sum, S_n , of the first n terms of an arithmetic sequence is given by

$$S_n = \frac{n}{2}(a_1 + a_n),$$

in which a_1 is the first term and a_n is the n th term.

HW Day1: p. 1020-1021:

35 - 40 all, 45, 47, 69, 71

EXAMPLE 4 Finding the Sum of n Terms of an Arithmetic Sequence

Find the sum of the first 100 terms of the arithmetic sequence: 1, 3, 5, 7, ...

$d=2$

Arith. Seq. Sum:

$$a_n = a_1 + (n-1)d$$

$$S_n = \left(\frac{n}{2}\right)(a_1 + a_n)$$

$$a_{100} = 1 + (100-1)(2)$$

$$S_{100} = \left(\frac{100}{2}\right)(1 + 199)$$

$$a_{100} = 199$$

$$S_{100} = 10000$$

✓ **Check Point 4** Find the sum of the first 15 terms of the arithmetic sequence 3, 6, 9, 12, ... $d=3$

Arith Seq. Sum:

$$a_{15} = 3 + (15-1)(3)$$

$$S_{15} = \left(\frac{15}{2}\right)(3 + 45)$$

$$a_{15} = 45$$

$$S_{15} = 360$$

✓ **Check Point 4b**

A theater has 30 seats in the first row, 32 seats in the second row, increasing by 2 seats per row for a total of 26 rows. How many seats are there in the theater?

$d=2$ $n=26$
 $a_1=30$

Arith. Seq. Sum:

$$a_{26} = 30 + (26-1)(2)$$

$$S_{26} = \left(\frac{26}{2}\right)(30 + 80)$$

$$a_{26} = 80$$

$$S_{26} = 1430 \text{ seats}$$

EXAMPLE 5 Using S_n to Evaluate a Summation

Find the following sum: $\sum_{i=1}^{25} (5i - 9)$. n = 25 terms

1st term: $a_1 = 5(1) - 9 = -4$ $S_{25} = \left(\frac{25}{2}\right)(-4 + 116)$

25th term: $a_{25} = 5(25) - 9 = 116$ $S_{25} = 1400$

Check Point 5 Find the following sum: $\sum_{i=1}^{30} (6i - 11)$.

$a_1 = 6(1) - 11 = -5$

$a_{30} = 6(30) - 11 = 169$

$S_{30} = \left(\frac{30}{2}\right)(-5 + 169)$

$S_{30} = 2460$

Last Example: Given the following, find the sum.

7, 18, 29, 40, ..., ..., 205.

$d = 11$

Arith. Seq.

$a_n = a_1 + (n-1)d$

$205 = 7 + (n-1)(11)$

$198 = 11n - 11$

$209 = 11n$

$n = 19$

Sum:

$S_{19} = \left(\frac{19}{2}\right)(7 + 205)$

$S_{19} = 2014$

Check your answers!

Answers to 3.2 Day 1

- | | |
|-------------|---------------------------|
| 35.) 1220 | 45.) 816 |
| 36.) 3775 | 47.) -1245 |
| 37.) 4400 | 69.) Company A: \$307,000 |
| 38.) 6600 | Company B: \$324,000 |
| 39.) 5050 | 71.) 2869 total seats |
| 40.) 10,100 | |