November 25, 2019

3.2: - Summations and Series - Day 2

The Sum of the First n Terms of a Geometric Sequence

The sum, S_n, of the first n terms of a geometric sequence is given by

$$\boxed{S_n = \frac{a_1(1 - r^n)}{1 - r},}$$

in which a_1 is the first term and r is the common ratio ($r \neq 1$).

HW Day2: p. 1035:
25 - 30 all,
37, 38, 41,
71, 74, 95, 96

EXAMPLE Finding the Sum of the First n Terms of a Geometric Sequence

Find the sum of the first 12 terms of the geometric sequence: 2, −8, 32, −128, ...

$n = 12$

$a_1 = 2$ $r = -4$

$$S_{12} = \frac{(2(1-(-4)^{12}))}{(1--4)}$$

$$= \boxed{-6,710,886}$$

Check Point Find the sum of the first nine terms of the geometric sequence: 2, −6, 18, −54, ...

$$S_9 = \frac{(2(1-(-3)^9))}{(1--3)} = \boxed{9842}$$

Check Point A job pays a salary of \$30,000 the first year. During the next 29 years, the salary increases by 6% each year. What is the total lifetime salary over the 30-year period?

$n = 30$ $a_1 = 30000$ $r = 1.06$

$$S_{30} = \frac{(30000(1-(1.06)^{30}))}{(1-1.06)}$$

$$= \boxed{\$2,371,746}$$

The Sum of an Infinite Geometric Series

If $-1 < r < 1$ (equivalently, $|r| < 1$), then the sum of the infinite geometric series

$$a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots,$$

in which a_1 is the first term and r is the common ratio, is given by

$$\boxed{S = \frac{a_1}{1 - r}.}$$

If $|r| \geq 1$, the infinite series does not have a sum.

EXAMPLE Find the sum of the infinite geometric series: $12 + 3 + \frac{3}{4} + \frac{3}{16} + \ldots$

a_1

$$S = \frac{12}{(1-(1/4))} = \boxed{16}$$

$r = \frac{3}{12}$

$r = 1/4$

Check Point Find the sum of the infinite geometric series: $3 + 2 + \frac{4}{3} + \frac{8}{9} + \cdots$.

$r = 2/3$

$$S = \frac{3}{(1-(2/3))} = \boxed{9}$$

Last Example... Find the sum of this finite geometric series.

$$3 + 6 + 12 + \ldots + 786432$$

$a_1 = 3$ $\quad r = 2$ $\quad n = ?$

Geom. Seq.

$a_n = a_1 (r)^{n-1}$

$786432 = 3(2)^{n-1}$

$262144 = (2)^{n-1}$

$\log_2(262144) = n - 1$

$18 = n - 1$

$19 = n$

Sum:

$$S_{19} = \frac{(3(1-(2)^{19}))}{(1-2)}$$

$$\boxed{= 1572861}$$

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Answers to 3.2 Day 2

25.) 531,440

26.) 12,285

27.) 2,049

28.) 177,148

29.) 8,191.5

30.) 5461/24

37.) 3/2

38.) 4/3

41.) 2/3

71.) $ 32,767

74.) Company A: $169,113

Company B: $169,892

95.) 30 days: $10,737,418.23

96.) Geometrically if over 30 days an exponent of 30 would yield a huge increase.