

Topic 3.3 - Conic Sections

Day 1 - Ellipses

HW: p. 930:
#s 1 - 7 odds, 11 - 17, odds 20, 21

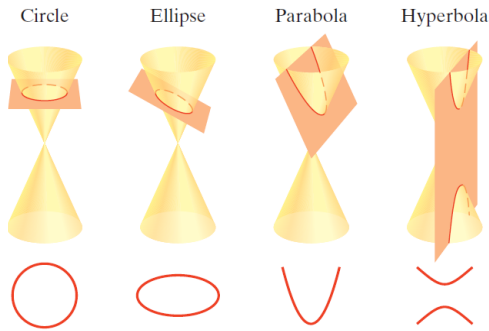


FIGURE 9.1 Obtaining the conic sections by intersecting a plane and a cone

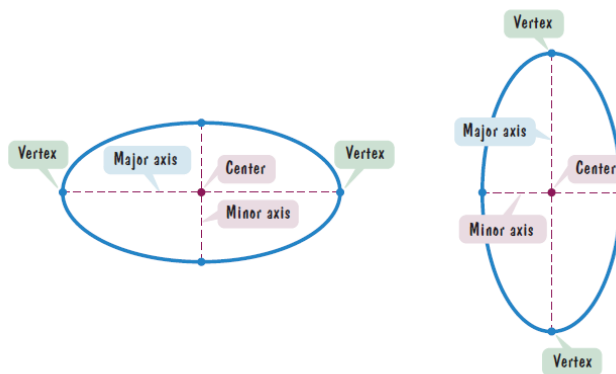


FIGURE 9.4 Horizontal and vertical elongations of an ellipse

Definition of an Ellipse

An **ellipse** is the set of all points, P , in a plane the sum of whose distances from two fixed points, F_1 and F_2 , is constant (see **Figure 9.3**). These two fixed points are called the **foci** (plural of **focus**). The midpoint of the segment connecting the foci is the **center** of the ellipse.

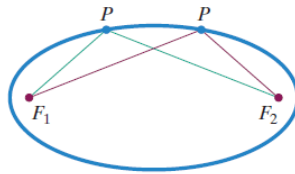


FIGURE 9.3

Standard Forms of the Equations of an Ellipse

The **standard form of the equation of an ellipse** with center at the origin, and major and minor axes of lengths $2a$ and $2b$ (where a and b are positive, and $a^2 > b^2$) is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Figure 9.6 illustrates that the vertices are on the major axis, a units from the center. The foci are on the major axis, c units from the center. For both equations, $b^2 = a^2 - c^2$. Equivalently, $c^2 = a^2 - b^2$.

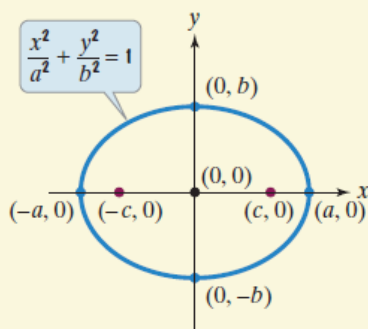


FIGURE 9.6(a) Major axis is horizontal with length $2a$.

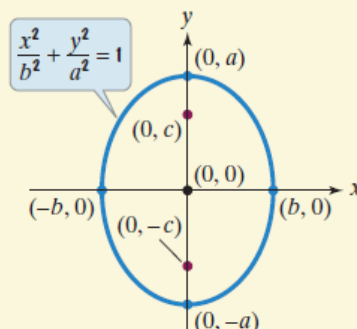


FIGURE 9.6(b) Major axis is vertical with length $2a$.

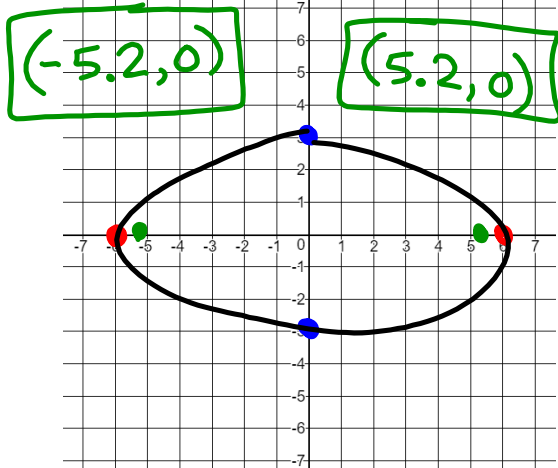
EXAMPLE 1 Graphing an Ellipse Centered at the Origin

Graph and locate the foci: $\frac{x^2}{36} + \frac{y^2}{9} = 1$

major axis (x): $a^2 = 36, a = 6$

minor axis (y): $b^2 = 9, b = 3$

foci: $c^2 = 36 - 9$
 $c^2 = 27, c = 5.2$

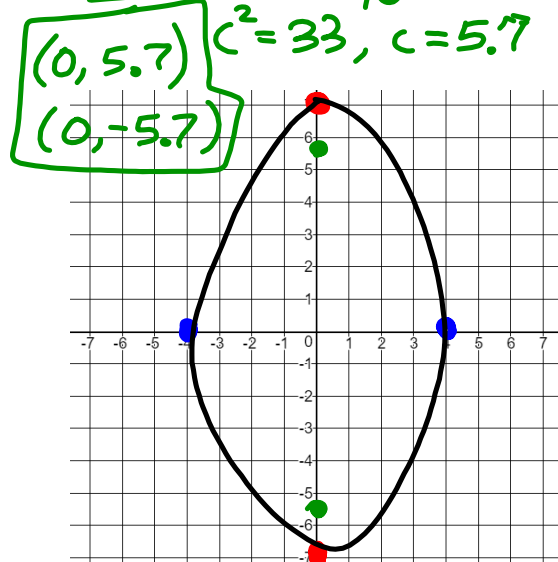


Graph and locate the foci: $\frac{x^2}{16} + \frac{y^2}{49} = 1$

major (y): $a^2 = 49, a = 7$

minor (x): $b^2 = 16, b = 4$

foci: $c^2 = 49 - 16$
 $c^2 = 33, c = 5.7$



EXAMPLE 2 Graphing an Ellipse Centered at the Origin

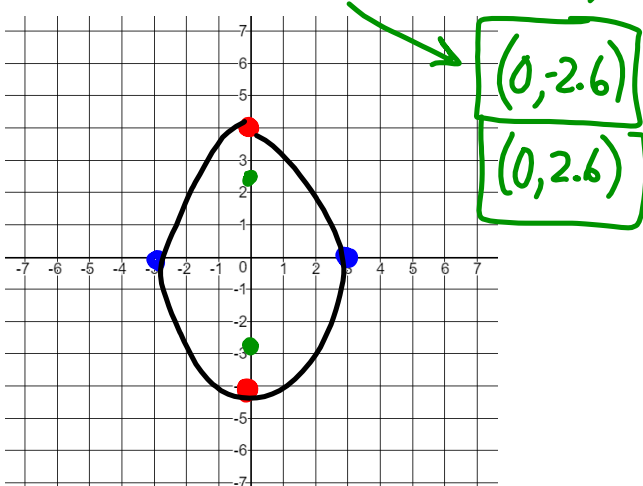
Graph and locate the foci: $\frac{16x^2}{144} + \frac{9y^2}{144} = \frac{144}{144}$

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

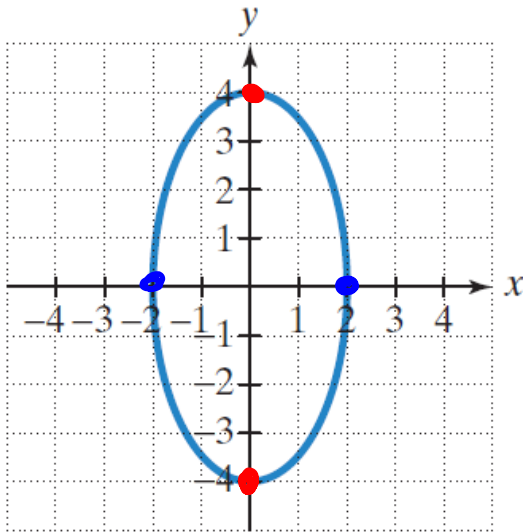
major y: $a^2 = 16, a = 4$

minor x: $b^2 = 9, b = 3$

foci: $c^2 = 16 - 9, c^2 = 7, c = 2.6$



Last Examples: Find the standard form equation of this ellipse. Then find the location of the foci.



$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

Major (y): $a=4, a^2=16$

Minor (x): $b=2, b^2=4$

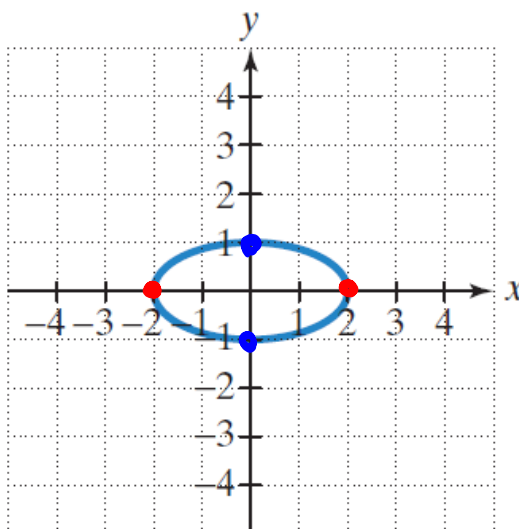
$$c^2 = 16 - 4$$

$$c^2 = 12$$

$$c \approx 3.5$$

$$\boxed{(0, -3.5) \text{ Foci}} \\ \boxed{(0, 3.5)}$$

Last Examples: Find the standard form equation of this ellipse. Then find the location of the foci.



$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

major (x): $a=2$
 $a^2=4$

minor (y): $b=1$
 $b^2=1$

foci → $\boxed{(1.7, 0) \text{ } (-1.7, 0)}$

$$c^2 = 4 - 1$$

$$c^2 = 3, c = 1.7$$