November 18, 2019

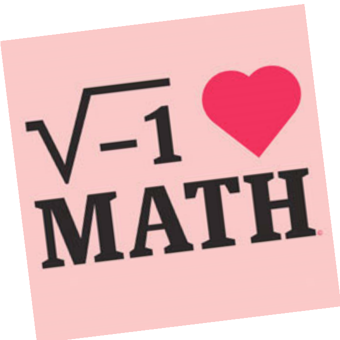
3.3 - Complex Quadratic Roots - Day 1

Imaginary and Complex Numbers

In previous courses, you probably learned that no value exists for the square roots of negative numbers. However, you can express values for these roots using **complex numbers**. Complex numbers have a real term and an imaginary term in the form $a + bi$, with a and b being real numbers and i being the **imaginary number** $\sqrt{-1}$.

HW Day 1 - 3.3 wksht.

The imaginary unit, i, is the complex number whose square is -1,



$$\boxed{i = \sqrt{-1}}$$

$$\downarrow$$

$$i^2 = -1$$

Powers of i

There is a pattern when the powers of i are evaluated.

The pattern continues. Every power of i that is a multiple of 4 will equal 1.

$$i = \sqrt{-1}$$
$$i^2 = -1$$
$$i^3 = i^2 \cdot i = -1 \cdot i = -i$$
$$i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$$

$$i^5 = i^4 \cdot i = 1 \cdot i = i$$
$$i^6 = i^4 \cdot i^2 = 1 \cdot -1 = -1$$
$$i^7 = i^4 \cdot i^3 = 1 \cdot -i = -i$$
$$i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1$$

Ex1.) Evaluate.

a.) i^{18}

$i^{16} \cdot i^2$

$(1) \cdot (-1) = \boxed{-1}$

b.) i^{27}

$i^{24} \cdot i^3$

$(1)(-i) = \boxed{-i}$

Evaluating Expressions with Imaginary Numbers

Expressions that contain no **real number** value can now be rewritten using imaginary number values.

Ex2.) Simplify.

$\sqrt{-7}$

$\boxed{i\sqrt{7}}$

$\sqrt{-36}$

$i\sqrt{36}$

$\boxed{6i}$

$\sqrt{-20}$

$i\sqrt{20}$

$i\sqrt{4 \cdot 5}$

$\boxed{2i\sqrt{5}}$

$\sqrt{-10} \cdot \sqrt{-5}$

$i\sqrt{10} \cdot i\sqrt{5}$

$i^2\sqrt{50}$

$-1\sqrt{25 \cdot 2}$

$\boxed{-5\sqrt{2}}$

$\sqrt{-12} + \sqrt{-27}$

$i\sqrt{12} + i\sqrt{27}$

$i\sqrt{4 \cdot 3} + i\sqrt{9 \cdot 3}$

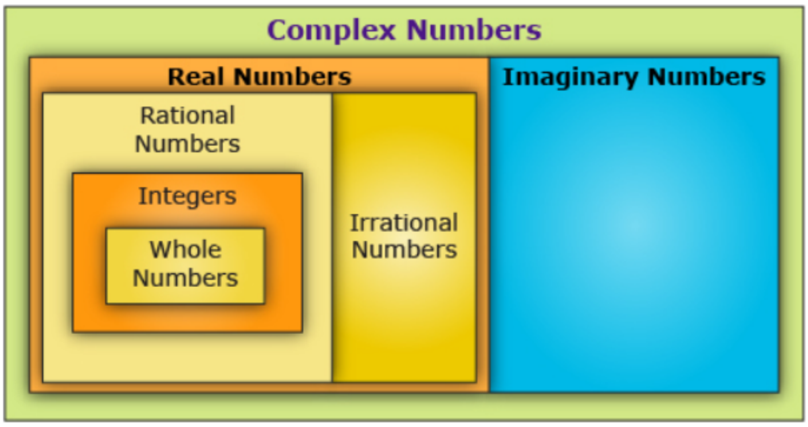
$2i\sqrt{3} + 3i\sqrt{3}$

$\boxed{5i\sqrt{3}}$

Complex Numbers

Complex numbers contain both real and imaginary numbers and are in the form $a + bi$. If the **complex number** is real, then $b = 0$. If the complex number is imaginary, then $a = 0$.

A **complex number** is a number in the form $a + bi$, where a and b are real numbers and i is $\sqrt{-1}$. *ex.)* $5 + 3i$



Operations with Complex Numbers

Note: As you perform operations with complex numbers, the terms and factors may need the commutative and associative properties applied to result in the proper final form of a complex number: $a + bi$.

Ex3.) Perform the indicated operations and simplify.

$(3+5i)+(1-2i)$

add like terms!

$\boxed{4+3i}$

$(8-9i)-(5-3i)$

$8-9i-5+3i$

$\boxed{3-6i}$

Operations with Complex Numbers

Ex4.) Perform the indicated operations and simplify.

$5i(7-2i)$

$i^2=-1$

$35i-10i^2$

$35i+10$

$\boxed{10+35i}$

$(5+3i)(5-3i)$

$i^2=-1$

$25-\cancel{15i}+\cancel{15i}-9i^2$

$25+9$

$\boxed{34}$

Operations with Complex Numbers

Ex5.) Perform the indicated operations and simplify.

$$\frac{8}{6i} \cdot \frac{i}{i} = \frac{8i}{6i^2}$$

$$\frac{8i}{-6} \rightsquigarrow \text{simplify} = \boxed{-\frac{4i}{3}}$$

$$\frac{6}{(3-5i)} \cdot \frac{(3+5i)}{(3+5i)}$$

$$\frac{18+30i}{9+15i-15i-25i^2}$$

$$\frac{18+30i}{34} \begin{matrix} \div 2 \quad \div 2 \\ \div 2 \end{matrix} \rightsquigarrow \boxed{\frac{9+15i}{17}}$$

Operations with Complex Numbers

Ex6.) Perform the indicated operation and simplify.

$$\frac{(8+2i)}{(-3+2i)} \cdot \frac{(-3-2i)}{(-3-2i)} = \frac{-24-16i-6i-4i^2}{9+6i-6i-4i^2}$$

$$\frac{-24-22i+4}{9+4} = \boxed{\frac{-20-22i}{13}}$$

Answers to HW 3.3 - Day 1 - Complex #'s

1.) $2i$ 2.) $9i$ 3.) $i\sqrt{7}$ 4.) $2i\sqrt{5}$ 5.) $4i\sqrt{3}$

6.) -1   7.) $-i$ 8.) i 9.) $1 - 7i$ 10.) $10 + 6i$

11.) -36   12.) $6 + 10i$ 13.) $288i$ 14.) $9 + 58i$

15.) $1 + 31i$ 16.) $7 - i$ 17.) $(2 + 3i) / -5$

18.) $(8 + 12i) / 13$ 19.) $(16 + 13i) / 17$