

3.3 - Complex Quadratic Roots - Day 2

Solutions to Quadratic Equations

Definition

a univariate polynomial equation of the second degree having the general form $ax^2 + bx + c = 0$, where a , b , and c are real numbers and $a \neq 0$

Any equation in which the highest exponent of the variables is two is known as a quadratic equation. Trajectories of a ball or a missile can be described using a quadratic equation. The formulas for finding the area of basic shapes such as squares or circles are also quadratic equations.

Ex1.) Solve each quadratic equation.

$$\begin{array}{l}
 -10x = -x^2 - 24 \\
 x^2 - 10x + 24 = 0 \\
 \text{Factor.} \\
 (x-6)(x-4) = 0 \\
 \boxed{x=6} \quad \boxed{x=4}
 \end{array}
 \qquad
 \begin{array}{l}
 \frac{3x^2}{3} = \frac{60}{3} \\
 \sqrt{x^2} = \sqrt{20} \\
 x = \pm\sqrt{20} \\
 \boxed{x=2\sqrt{5}} \quad \boxed{x=-2\sqrt{5}}
 \end{array}
 \qquad
 \begin{array}{l}
 x^2 + 25 = 0 \\
 \sqrt{x^2} = \sqrt{-25} \\
 x = \pm 5i \\
 \boxed{x=5i} \\
 \boxed{x=-5i}
 \end{array}$$

Ex2.) Solve this quadratic equation.

$$\begin{array}{l}
 x^2 + 3x + 5 = 0 \quad a=1 \\
 \qquad \qquad \qquad b=3 \\
 \qquad \qquad \qquad c=5 \\
 x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(5)}}{2(1)} \quad \text{— calc.} \\
 x = \frac{-3 \pm \sqrt{-11}}{2} \rightarrow \boxed{x = \frac{-3 \pm i\sqrt{11}}{2}}
 \end{array}$$

Quadratic Formula

*When you cannot factor or solve straight up...

Given a quadratic equation in standard form, $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ex3.) Solve this quadratic equation.

$$x^2 + 5 = 4x \quad a=1, b=-4, c=5$$

$$x^2 - 4x + 5 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)} \quad \text{calc. } 15x$$

$$x = \frac{4 \pm \sqrt{-4}}{2} \rightarrow x = \frac{4 \pm 2i}{2}$$

divide by 2

$x = 2 + i$

$x = 2 - i$

Ex4.) Solve this quadratic equation.

$$2x^2 = 11x + 21 \quad a=2, b=-11, c=-21$$

$$2x^2 - 11x - 21 = 0$$

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(2)(-21)}}{2(2)} \quad \text{calc.}$$

$$x = \frac{11 \pm \sqrt{289}}{4} \rightarrow \frac{11 \pm 17}{4}$$

$x = \frac{(11+17)}{4} = \boxed{7}$

$x = \frac{(11-17)}{4} = \boxed{-\frac{3}{2}}$