

3.3 - Day 3 - Ellipses

Getting an Ellipse Equation into Standard Form by Completing the Square

HW: p. 931:

#'s 51 - 55 odds,

65,67

In some cases, it is necessary to convert the equation of an ellipse to standard form by completing the square on x and y . For example, suppose that we wish to graph the ellipse whose equation is

$$9x^2 + 4y^2 - 18x + 16y - 11 = 0.$$

Because we plan to complete the square on both x and y , we need to rearrange terms so that

- x -terms are arranged in descending order.
- y -terms are arranged in descending order.
- the constant term appears on the right.

Complete the Square:

$$\left(\frac{b}{2}\right)^2$$

Ex. Convert the equation into standard form. Then graph and find the foci. (graph on next slide)

$$x^2 + 4y^2 + 10x - 8y + 13 = 0$$

$$\begin{aligned}
 &x^2 + 10x + 4y^2 - 8y = -13 \\
 &\boxed{x^2 + 10x + 25} + 4\boxed{(y^2 - 2y + 1)} = -13 + \underline{25} + \underline{4} \\
 &\quad \begin{array}{l} \text{Factor} \nearrow \\ \left(\frac{10}{2}\right)^2 \nearrow \end{array} \quad \begin{array}{l} \text{Factor} \nearrow \\ \left(\frac{-2}{2}\right)^2 \nearrow \end{array} \\
 &\frac{(x+5)^2}{16} + \frac{4(y-1)^2}{16} = \frac{16}{16} \\
 &\boxed{\frac{(x+5)^2}{16} + \frac{(y-1)^2}{4} = 1}
 \end{aligned}$$

EXAMPLE 5 An Application Involving an Ellipse

A semielliptical archway over a one-way road has a height of 10 feet and a width of 40 feet (see **Figure 9.11**). Your truck has a width of 10 feet and a height of 9 feet. Will your truck clear the opening of the archway?

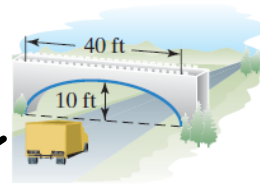
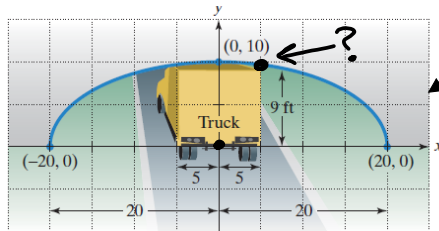


FIGURE 9.11
A semielliptical archway



$$\frac{x^2}{400} + \frac{y^2}{100} = 1$$

$$a=20 \quad b=10$$

$$a^2=400 \quad b^2=100$$

Plug in $x=5$

$$\frac{5^2}{400} + \frac{y^2}{100} = 1$$

$$.0625 + \frac{y^2}{100} = 1$$

$$-.0625 \quad \quad \quad -.0625$$

So, at $x=5$ ft.
From center the
truck will clear
the bridge.

$$100 \cdot \frac{y^2}{100} = 0.9375 \cdot 100$$

$$\sqrt{y^2} = \sqrt{93.75}$$

$$y = 9.68 \text{ ft.}$$