December 12, 2019

3.3 - Day 4 - Hyperbolas

HW: p. 945:
#'s 1 - 4 all,
13, 17, 21, 23

The **standard form of the equation of a hyperbola** with center at the origin is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Figure 9.16(a) illustrates that for the equation on the left, the transverse axis lies on the x-axis. **Figure 9.16(b)** illustrates that for the equation on the right, the transverse axis lies on the y-axis. The vertices are a units from the center and the foci are c units from the center. For both equations, $b^2 = c^2 - a^2$. Equivalently, $c^2 = a^2 + b^2$.

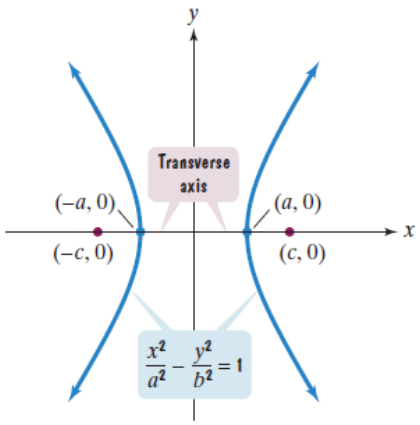


FIGURE 9.16(a) Transverse axis lies on the x-axis.

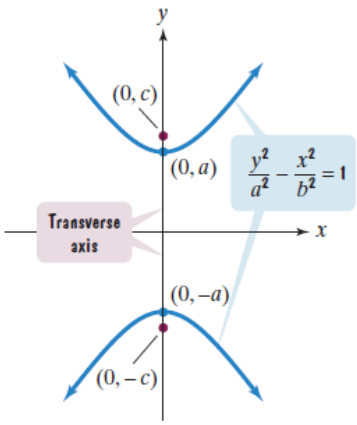
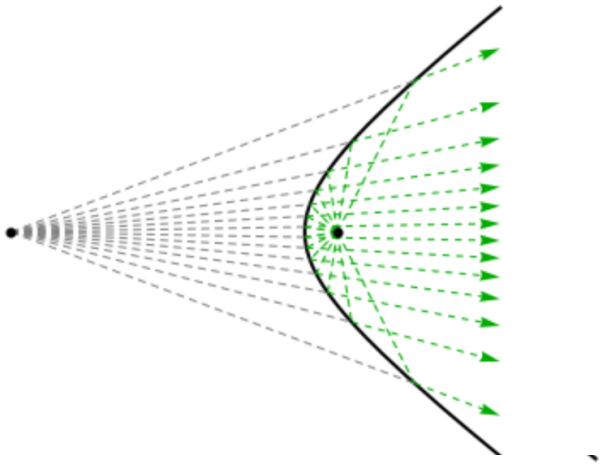


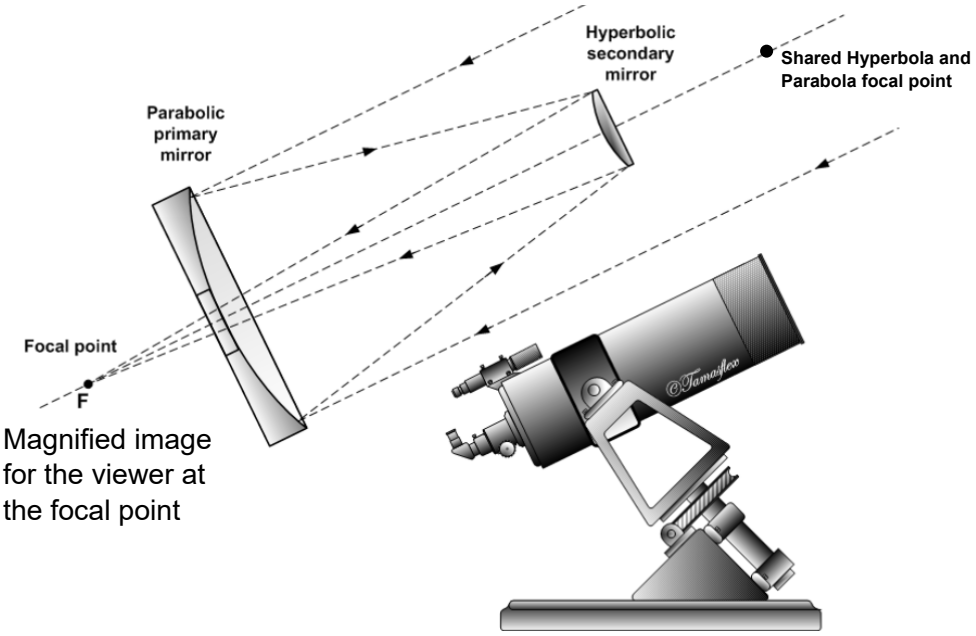
FIGURE 9.16(b) Transverse axis lies on the y-axis.

Reflective Properties



Any sound or light wave leaving one focus will reflect off the hyperbola in the same direction as if it was leaving the other focus.

Uses: Street lights, musical instruments, cooling towers…



EXAMPLE 1 Finding Vertices and Foci from a Hyperbola's Equation

Find the vertices and locate the foci for each of the following hyperbolas with the given equation:

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

$b^2 = 9,\ b = 3$

$c^2 = a^2 + b^2$

$c^2 = 16 + 9$

$c^2 = 25$

$c = 5$

Transverse axis is x-axis

$a^2 = 16$

$a = 4$

Vertices: (4,0) (-4,0)

Foci: (5,0) (-5,0)

Check Point 1 Find the vertices and locate the foci for each of the following hyperbolas with the given equation:

$$\frac{y^2}{25} - \frac{x^2}{16} = 1$$

$b^2 = 16$

$c^2 = 25 + 16$

$c^2 = 41$

$c \approx 6.4$

Transverse axis is y-axis.

Foci: $(0, 6.4)$ $(0, -6.4)$

$a^2 = 25$

$a = 5$

Vertices: $(0, 5)$ $(0, -5)$

Asymptotes of a Hyperbola

Figure 9.20 shows the asymptotes for the graphs of hyperbolas centered at the origin. The asymptotes pass through the corners of a rectangle. Note that the dimensions of this rectangle are $2a$ by $2b$. The line segment of length $2b$ is the **conjugate axis** of the hyperbola and is perpendicular to the transverse axis through the center of the hyperbola.

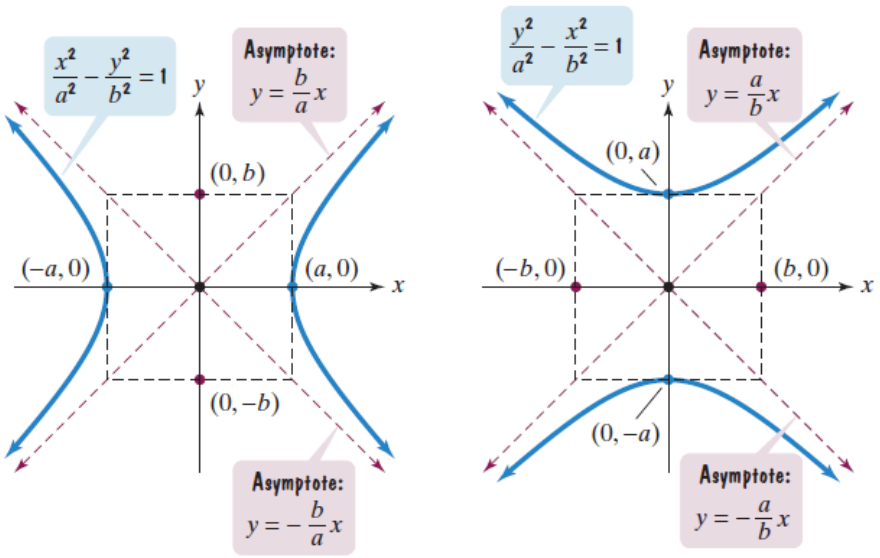


FIGURE 9.20 Asymptotes of a hyperbola

Graphing Hyperbolas Centered at the Origin

Hyperbolas are graphed using vertices and asymptotes.

Graphing Hyperbolas

1. Locate the vertices.
2. Use dashed lines to draw the rectangle centered at the origin with sides parallel to the axes, crossing one axis at $\pm a$ and the other at $\pm b$.
3. Use dashed lines to draw the diagonals of this rectangle and extend them to obtain the asymptotes.
4. Draw the two branches of the hyperbola by starting at each vertex and approaching the asymptotes.

EXAMPLE Graphing a Hyperbola

Graph and locate the foci: $9y^2 - 4x^2 = 36$. What are the equations of the asymptotes?

$\frac{9y^2}{36} - \frac{4x^2}{36} = \frac{36}{36}$

$\frac{y^2}{4} - \frac{x^2}{9} = 1$

Foci:
(0, 3.6)
(0, -3.6)

$b^2 = 9$

$b = 3$

Transverse y-axis

$a^2 = 4$

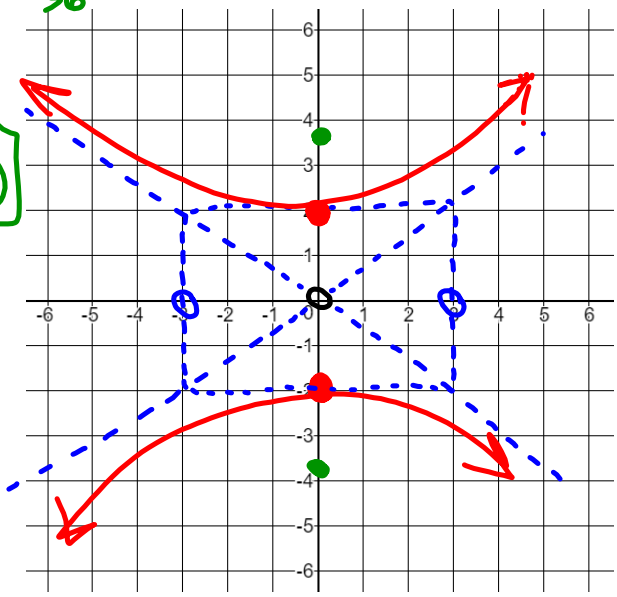
$a = 2$

Asymptotes:

$y = \pm\frac{2}{3}x$

$c^2 = 4 + 9$

$c^2 = 13, \quad c = 3.6$



Check Point 2

Graph and find the foci. Also, write the equations of the asymptotes.

$$\frac{x^2}{16} - \frac{4y^2}{16} = \frac{16}{16}$$

$$\frac{x^2}{16} - \frac{y^2}{4} = 1$$

$a^2 = 16$

$a = 4$

$b^2 = 4$

$b = 2$

Asymptotes

$y = \pm\frac{1}{2}x$

$c^2 = 16 + 4$

$c^2 = 20$

$c \approx 4.5$

Foci $(-4.5, 0)$ $(4.5, 0)$