4.1 - Systems of Linear Equations - Day 1

HW Day 1:
#'s: 4 - 28 evens

Systems of Linear Equations and Their Solutions

All equations in the form $Ax + By = C$ are straight lines when graphed. Two such equations are called a **system of linear equations** or a **linear system. A solution to a system of linear equations in two variables** is an ordered pair that satisfies both equations in the system. For example, (3, 4) satisfies the system

$$\begin{cases} x + y = 7 & \text{(3 + 4 is, indeed, 7.)} \\ x - y = -1. & \text{(3 − 4 is, indeed, −1.)} \end{cases}$$

Thus, (3, 4) satisfies both equations and is a solution of the system. The solution can be described by saying that $x = 3$ and $y = 4$. The solution can also be described using set notation. The solution set to the system is $\{(3, 4)\}$—that is, the set consisting of the ordered pair (3, 4).

A system of linear equations can have exactly one solution, no solution, or infinitely many solutions. We begin with systems that have exactly one solution.

Check Point 1 Consider the system:

$$\begin{cases} 2x - 3y = -4 \\ 2x + y = 4. \end{cases}$$

Determine if the ordered pair (7, 6) is a solution of the system:

x y

$2(7) - 3(6) = -4$

$14 - 18 = -4$

$-4 = -4$ ✓

NOT a Solution

$2(7) + (6) = 4$

$14 + 6 = 4$

$20 = 4$

NO!

We could graph by getting each equation into slope-intercept form...

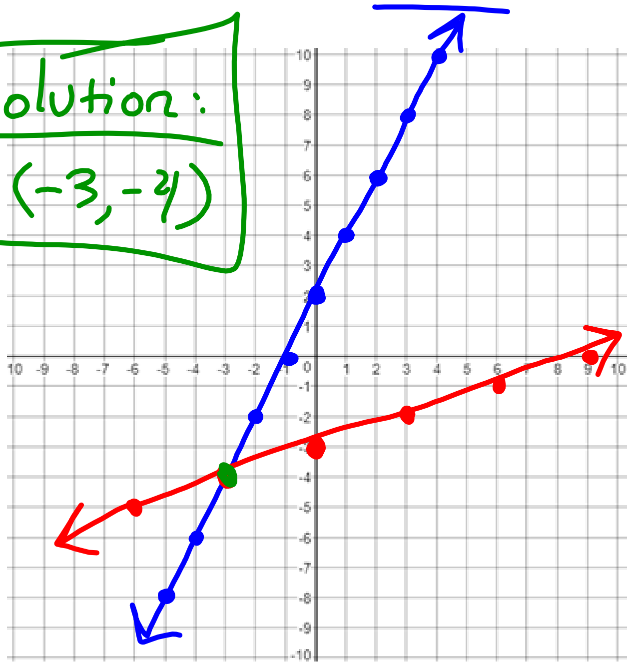
Consider the system:

$$\begin{cases} 2x = -2 + y \\ \dfrac{3y}{3} = \dfrac{x}{3} - \dfrac{9}{3} \end{cases}$$

$2x + 2 = y$

- $y = 2x + 2$
- $y = \frac{1}{3}x - 3$

Solution: (−3, −4)



Eliminating a Variable Using the Substitution Method

Finding the solution to a linear system by graphing equations may not be easy to do. For example, a solution of $\left(-\frac{2}{3}, \frac{157}{29}\right)$ would be difficult to "see" as an intersection point on a graph.

Let's consider a method that does not depend on finding a system's solution visually: the substitution method. This method involves converting the system to one equation in one variable by an appropriate substitution.

Solving Linear Systems by Substitution

1. Solve either of the equations for one variable in terms of the other. (If one of the equations is already in this form, you can skip this step.)
2. Substitute the expression found in step 1 into the *other* equation. This will result in an equation in one variable.
3. Solve the equation containing one variable.
4. Back-substitute the value found in step 3 into one of the original equations. Simplify and find the value of the remaining variable.
5. Check the proposed solution in both of the system's given equations.

EXAMPLE 2 Solving a System by Substitution

Solve by the substitution method:

$$\begin{cases} 5x - 4y = 9 \\ x - 2y = -3. \end{cases}$$

sub.

$+2y$

$x = 2y - 3$

$5(2y-3) - 4y = 9$

$10y - 15 - 4y = 9$

$+15 \qquad +15$

$6y = 24$

$y = 4$

$x = 2(4) - 3$

$x = 8 - 3$

$x = 5$

Solution: $\boxed{(5,4)}$

Check Point 2 Solve by the substitution method:

$$\begin{cases} 3x + 2y = 4 \\ 2x + y = 1. \end{cases}$$

sub.

$y = -2x + 1$

$3x + 2(-2x+1) = 4$

$3x - 4x + 2 = 4$

$-x + 2 = 4$

$-x = 2$

$x = -2$

$y = -2(-2) + 1$

$y = 4 + 1$

$y = 5$

$\boxed{(-2, 5)}$

Eliminating a Variable Using the Addition Method (Elimination Method)

The substitution method is most useful if one of the given equations has an isolated variable. A second, and frequently the easiest, method for solving a linear system is the addition method. Like the substitution method, the addition method involves eliminating a variable and ultimately solving an equation containing only one variable. However, this time we eliminate a variable by adding the equations.

For example, consider the following system of linear equations:

$$+\begin{cases} \cancel{3x} - 4y = 11 \\ \cancel{-3x} + 2y = -7. \end{cases}$$

$-2y = 4$

$y = -2$

$3x - 4(-2) = 11$

$3x + 8 = 11$

$3x = 3$

$x = 1$

Solution: $\boxed{(1, -2)}$

Solving Linear Systems by Addition (Elimination Method)

1. If necessary, rewrite both equations in the form $Ax + By = C$.
2. If necessary, multiply either equation or both equations by appropriate nonzero numbers so that the sum of the x-coefficients or the sum of the y-coefficients is 0.
3. Add the equations in step 2. The sum is an equation in one variable.
4. Solve the equation in one variable.
5. Back-substitute the value obtained in step 4 into either of the given equations and solve for the other variable.
6. Check the solution in both of the original equations.

EXAMPLE 3 Solving a System by the Addition Method

Solve by the addition method:

$$4\left(3x + 2y = 48\right) \qquad 12x + 8y = 192$$
$$9x - 8y = -24. \rightarrow \quad + \; 9x - 8y = -24$$

$$21x = 168$$
$$x = 8$$

$$3(8) + 2y = 48$$
$$24 + 2y = 48$$
$$2y = 24$$
$$y = 12$$

$$\boxed{(8, 12)}$$