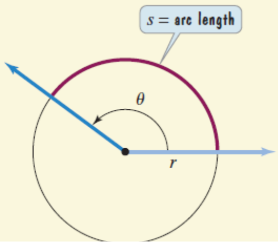
4.1 - Day 2 --- Angles and Radians

The Length of a Circular Arc

Let r be the radius of a circle and θ the nonnegative radian measure of a central angle of the circle. The length of the arc intercepted by the central angle is

$$s = r\theta.$$

must be in radians!



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EXAMPLE 8 Finding the Length of a Circular Arc

A circle has a radius of 10 inches. Find the length of the arc intercepted by a central angle of 120°.

$\theta = 120° \cdot \frac{\pi}{180} = \frac{2\pi}{3}$

$S = (10 \text{ in.})\left(\frac{2\pi}{3}\right)$

$\boxed{S = \frac{20\pi}{3} \text{ in.}}$, $\boxed{20.94 \text{ in.}}$

Check Point 8 A circle has a radius of 6 inches. Find the length of the arc intercepted by a central angle of 45°. Express arc length in terms of π. Then round your answer to two decimal places.

$\theta = 45° \rightsquigarrow \frac{\pi}{4}$

$S = (6 \text{ in.})\left(\frac{\pi}{4}\right) = \boxed{\frac{3\pi}{2} \text{ in.}}$, $\boxed{4.71 \text{ in.}}$

Linear and Angular Speed

A carousel contains four circular rows of animals. As the carousel revolves, the animals in the outer row travel a greater distance per unit of time than those in the inner rows. These animals have a greater *linear speed* than those in the inner rows. By contrast, all animals, regardless of the row, complete the same number of revolutions per unit of time. All animals in the four circular rows travel at the same *angular speed*.

Using v for linear speed and ω (omega) for angular speed, we define these two kinds of speed along a circular path as follows:

Definitions of Linear and Angular Speed

If a point is in motion on a circle of radius r through an angle of θ radians in time t, then its **linear speed** is

$$v = \frac{s}{t},$$

where s is the arc length given by $s = r\theta$, and its **angular speed** is

$$\boxed{\omega = \frac{\theta}{t}.}$$

Linear Speed in Terms of Angular Speed

The linear speed, v, of a point a distance r from the center of rotation is given by

$$\boxed{v = r\omega,}$$

where ω is the angular speed in radians per unit of time.

EXAMPLE 9 Finding Linear Speed

A wind machine used to generate electricity has blades that are 10 feet in length (see **Figure 4.18**). The propeller is rotating at four revolutions per second. Find the linear speed, in feet per second, of the tips of the blades.

$r = 10 \text{ ft.}$

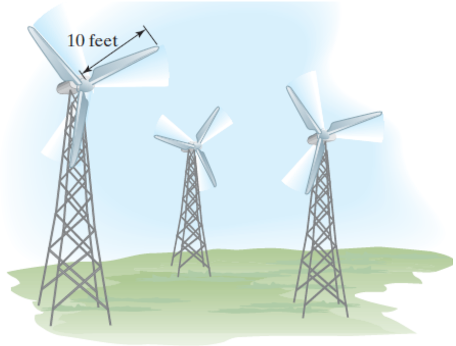

FIGURE 4.18

$$\omega = \frac{4 \text{ revs.}}{1 \text{ sec.}} \cdot \frac{2\pi}{1 \text{ rev.}} = 8\pi/\text{sec}$$

$$V = (10 \text{ ft.})(8\pi/\text{sec}) = \boxed{80\pi \text{ ft./sec}}$$

$\boxed{251.33 \text{ ft./sec}}$

Ex. A Ferris wheel has a radius of 25 feet. The wheel is rotating at two revolutions per minute. Find the linear speed, in feet per minute, of a seat on this Ferris wheel.

$$\omega = \frac{2 \text{ revs.}}{1 \text{ min.}} \cdot \frac{2\pi}{1 \text{ rev.}} = 4\pi/\text{min}$$

$\boxed{314.16 \text{ ft./min}}$

$$V = (25 \text{ ft.})(4\pi/\text{min}) = \boxed{100\pi \text{ ft./min}}$$

Ex. The minute hand of a clock is 6 inches long and moves from 12 to 4 o'clock. How far does the tip of the minute hand move? Express your answer in terms of π and then round to two decimal places.

$r = 6 \text{ in.}$ $\quad 120°\left(\frac{\pi}{180}\right)$

$$S = (6 \text{ in.})\left(\frac{2\pi}{3}\right)$$

$$S = \boxed{4\pi \text{ in.}}$$

$$\boxed{\approx 12.57 \text{ in.}}$$