4.1 - Systems of Linear Equations (More Apps) -- Day 3

Chemists and pharmacists often have to change the concentration of solutions and other mixtures. In these situations, the amount of a particular ingredient in the solution or mixture is expressed as a percentage of the total.

HW Day 3:
#'s: 55, 57, 61, 64
67 - 71 all, 73, 75

EXAMPLE 7 Solving a Mixture Problem

A chemist working on a flu vaccine needs to mix a 10% sodium-iodine solution with a 60% sodium-iodine solution to obtain 50 milliliters of a 30% sodium-iodine solution. How many milliliters of the 10% solution and of the 60% solution should be mixed?

x y (50x.30)

mL $(x + y = 50) \cdot -.10$

% $.10x + .60y = 15$

$+ \quad -.10x - .10y = -5$

$.50y = 10$

$\boxed{y = 20 \text{ mL of } 60\% \text{ solution}}$

$x + 20 = 50$

$\boxed{x = 30 \text{ mL of } 10\% \text{ solution}}$

Check Point 7 A chemist needs to mix an 18% acid solution with a 45% acid solution to obtain 12 liters of a 36% acid solution. How many liters of each of the acid solutions must be used?

$12 \times .36$

liters: $x + y = 12$

% : $.18x + .45y = 4.32$

Solve using elimination or substitution...

$\boxed{\begin{array}{l} x = 4 \text{ liters of } 18\% \text{ solution} \\ y = 8 \text{ liters of } 45\% \text{ solution} \end{array}}$

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We have seen that if an object moves at an average velocity v, the distance, s, covered in time t is given by the formula

$d = rt$

$$s = vt \quad \text{Distance equals velocity times time.}$$

Recall that objects that move in accordance with this formula are said to be in **uniform motion**. Wind and water current have the effect of increasing or decreasing a traveler's velocity.

EXAMPLE 8 Solving a Uniform Motion Problem

$d = rt$
$r = d/t$

When a small airplane flies with the wind, it can travel 450 miles in 3 hours. When the same airplane flies in the opposite direction against the wind, it takes 5 hours to fly the same distance. Find the average velocity of the plane in still air and the average velocity of the wind.

x y

$$\begin{cases} x + y = 150 \leftarrow \frac{450 \text{ mi}}{3 \text{ hr.}} \\ x - y = 90 \leftarrow \frac{450 \text{ mi}}{5 \text{ hr.}} \end{cases}$$

$+$

$2x = 240$

$\boxed{x = 120 \text{ mi/hr. plane}}$

$120 + y = 150$

$\boxed{y = 30 \text{ mi/hr. wind}}$

Check Point 8 With the current, a motorboat can travel 84 miles in 2 hours. Against the current, the same trip takes 3 hours. Find the average velocity of the boat in still water and the average velocity of the current.

$$\begin{cases} x + y = 42 \leftarrow 84/2 \\ x - y = 28 \leftarrow 84/3 \end{cases}$$

Solve......

$\boxed{x = 35 \text{ mi/hr. boat} \quad y = 7 \text{ mi/hr. current}}$

Functions of Business: Break-Even Analysis

Suppose that a company produces and sells x units of a product. Its *revenue* is the money generated by selling x units of the product. Its *cost* is the cost of producing x units of the product.

Revenue and Cost Functions

A company produces and sells x units of a product.

Revenue Function

$$R(x) = (\text{price per unit sold})x$$

Cost Function

$$C(x) = \text{fixed cost} + (\text{cost per unit produced})x$$

The point of intersection of the graphs of the revenue and cost functions is called the **break-even point**. The x-coordinate of the point reveals the number of units that a company must produce and sell so that money coming in, the revenue, is equal to money going out, the cost. The y-coordinate of the break-even point gives the amount of money coming in and going out. Example 9 illustrates the use of the substitution method in determining a company's break-even point.

EXAMPLE 9 Finding a Break-Even Point

Technology is now promising to bring light, fast, and beautiful wheelchairs to millions of people with disabilities. A company is planning to manufacture these radically different wheelchairs. Fixed cost will be \$500,000 and it will cost \$400 to produce each wheelchair. Each wheelchair will be sold for \$600.

a. Write the cost function, C, of producing x wheelchairs.
b. Write the revenue function, R, from the sale of x wheelchairs.
c. Determine the break-even point. Describe what this means.
d. Write a function, P, which represents the profit.

$\boxed{a.)\ C(x) = 400x + 500000}$

$\boxed{b.)\ R(x) = 600x}$

$C(x) = R(x)$

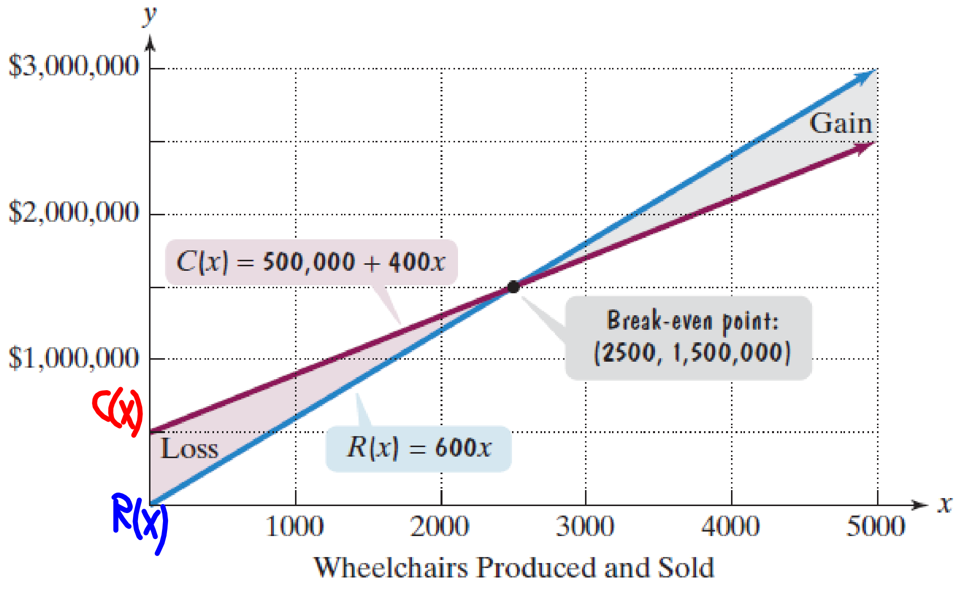
$400x + 500000 = 600x$

$500000 = 200x$

$x = 2500$

$\boxed{(2500, 1500000)}$

$600(2500)$



The Profit Function

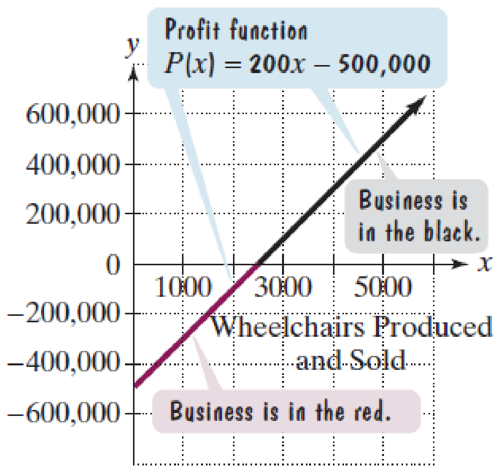
The profit, $P(x)$, generated after producing and selling x units of a product is given by the **profit function**

$$P(x) = R(x) - C(x),$$

where R and C are the revenue and cost functions, respectively.

The profit function for the wheelchair business in Example 7 is

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 600x - (500{,}000 + 400x) \\ &= 200x - 500{,}000. \end{aligned}$$



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#'s: 55, 57, 61, 64
67 - 71 all, 73, 75

HW DAY 3 - Answers:

55.) 100 gallons of 5% wine and 100 gallons of 9% wine.

57.) 96 grams of 18-karat (75%) gold and 204 grams of 12-karat (50%) gold.

61.) 130 mph plane in still air and a 30 mph wind.

64.) 21 mph boat in still water and a 3 mph water current.

67.) 500 radios must be sold to break-even.

68.) More than 500 radios must be produced/sold to profit.

69.) R(200) - C(200) = - $6000. This means they will lose $6000 if they only produce and sell 200 radios.

70.) R(300) - C(300) = - $4000. This means they will lose $4000 if they only produce and sell 300 radios.

71.) a.) Profit Function: $P(x) = 20x - 10000$

b.) If 10,000 radios are produced/sold, then profit will be $190,000.

73.) a.) $C(x) = 18000 + 20x$

b.) $R(x) = 80x$

c.) Break-Even: (300, 24000). The company must produce and sell 300 canoes to break-even at cost and revenue of $24,000.

75.) a.) $C(x) = 30000 + 2500x$

b.) $R(x) = 3125x$

c.) Break-Even: (48, 150000). The company must produce and sellout 48 performances to break-even at cost and revenue of $150,000.