December 12, 2019

4.1 - Systems of Linear Equations - Day 4

Systems of Linear Equations in Three Variables and Their Solutions

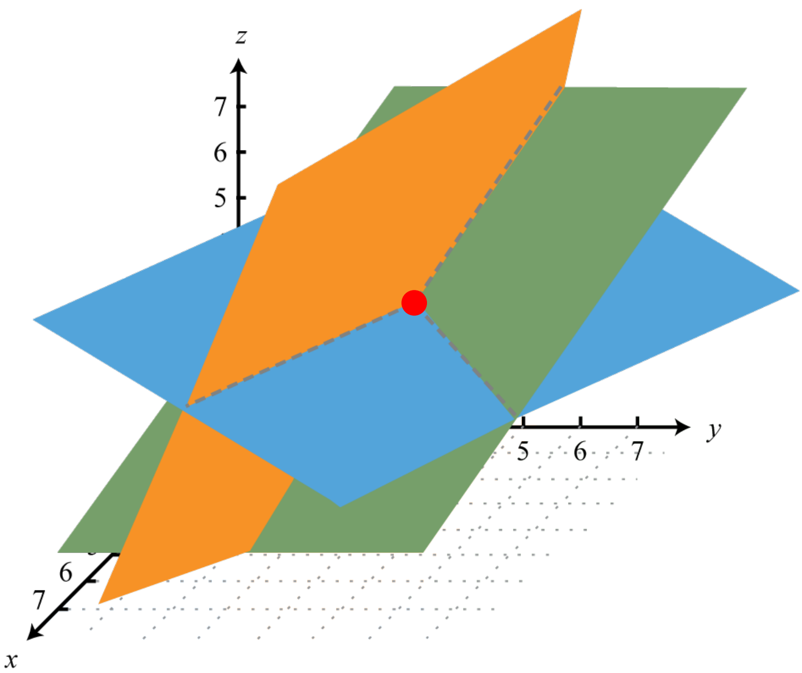
An equation such as $x + 2y - 3z = 9$ is called a *linear equation in three variables*. In general, any equation of the form

$$Ax + By + Cz = D,$$

where A, B, C, and D are real numbers such that A, B, and C are not all 0, is a **linear equation in three variables: x, y, and z.** The graph of this linear equation in three variables is a plane in three-dimensional space.

Solving Linear Systems in Three Variables by Eliminating Variables

1. Reduce the system to two equations in two variables. This is usually accomplished by taking two different pairs of equations and using the addition method to eliminate the same variable from both pairs.
2. Solve the resulting system of two equations in two variables using addition or substitution. The result is an equation in one variable that gives the value of that variable.
3. Back-substitute the value of the variable found in step 2 into either of the equations in two variables to find the value of the second variable.
4. Use the values of the two variables from steps 2 and 3 to find the value of the third variable by back-substituting into one of the original equations.
5. Check the proposed solution in each of the original equations.



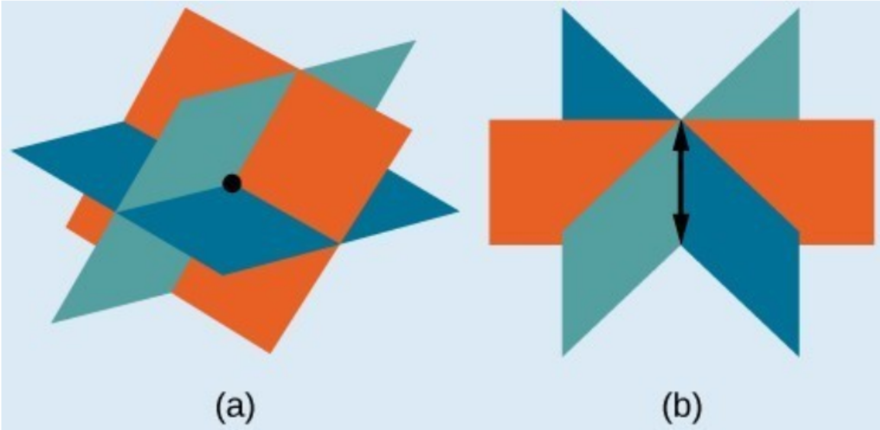


Figure 2. (a)Three planes intersect at a single point, representing a three-by-three system with a single solution. (b) Three planes intersect in a line, representing a three-by-three system with infinite solutions.

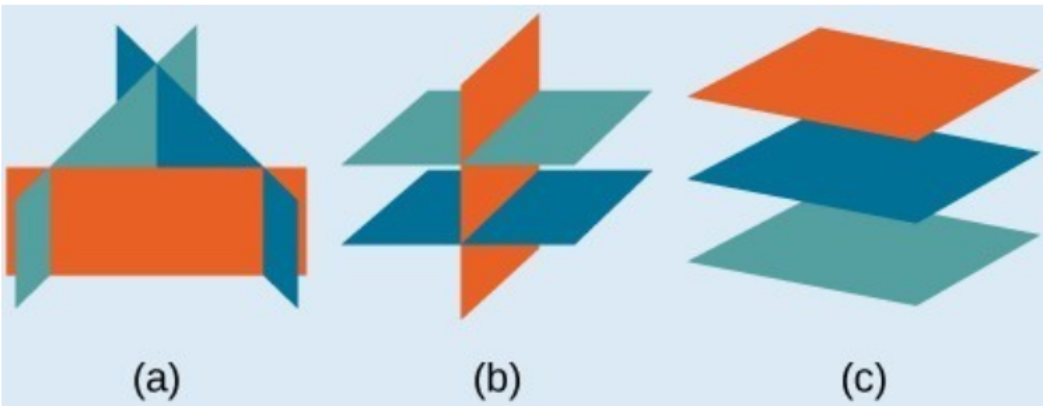


Figure 3 These are all systems with no solutions. They represent a system where there are no consistent set of x,y,z that satisfies all three equations in the system.

EXAMPLE 1 Solving a System in Three Variables

$$\begin{cases} x + y + z = 4 \\ x - 2y - z = 1 \\ 2x - y - 2z = -1 \end{cases}$$

$2(x + y + z = 4) \rightarrow 2x + 2y + 2z = 8$ ①

$2(x - 2y - z = 1) \rightarrow 2x - 4y - 2z = 2$ ②

$2x - y - 2z = -1 \rightarrow 2x - y - 2z = -1$ ③

①+②: $4x - 2y = 10$

①+③: $-1(4x + y = 7)$

$+ \quad -4x - y = -7$

$-3y = 3$, $y = -1$

$4x + (-1) = 7$

$4x = 8$

$x = 2$

$x + y + z = 4$

$2 - 1 + z = 4$

$z = 3$

$(2, -1, 3)$

Solution

Check Point 1

$$\begin{cases} 6x + 3y - 6z = 24 & ① \\ 5x - 5y - 6z = 19 & ② \\ -x + 4y + 6z = 1 & ③ \end{cases}$$

$-(5) + 4(0) + 6z = 1$

$6z = 6$, $z = 1$

①+③ : $5x + 7y = 25 \rightarrow 5x + 7y = 25$

②+③ : $7(4x - y = 20) \rightarrow 28x - 7y = 140$

$33x = 165$

$x = 5$

$4(5) - y = 20$

$20 - y = 20$

$-y = 0$

$y = 0$

$(5, 0, 1)$ solution

EXAMPLE 3 Solving a System of Equations with a Missing Term

Solve the system:

$$-2\begin{cases} x + z = 8 & \text{Equation 1} \\ (x + y + 2z = 17) & \text{Equation 2} \\ x + 2y + z = 16. & \text{Equation 3} \end{cases}$$

$-2x - 2y - 4z = -34$

$+\; x + 2y + z = 16$

$-x - 3z = -18$

$+\; x + z = 8$

$-2z = -10$

$z = 5$

$x + 5 = 8$

$x = 3$

$x + 2y + z = 16$

$3 + 2y + 5 = 16$

$2y = 8$

$y = 4$

solution

$(3, 4, 5)$