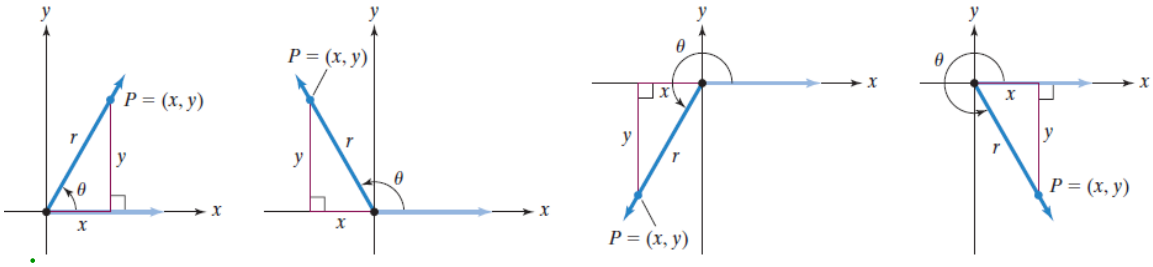
January 15, 2020

Topic #2 - Day 1 - 4.2 Trig. Functions of Any Angle



(a) θ lies in quadrant I. (b) θ lies in quadrant II. (c) θ lies in quadrant III. (d) θ lies in quadrant IV.

Definitions of Trigonometric Functions of Any Angle

Let θ be any angle in standard position and let $P = (x, y)$ be a point on the terminal side of θ. If $r = \sqrt{x^2 + y^2}$ is the distance from $(0, 0)$ to (x, y), as shown in **Figure 4.41** on the previous page, the **six trigonometric functions of θ** are defined by the following ratios:

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}, \; y \neq 0$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}, \; x \neq 0$$

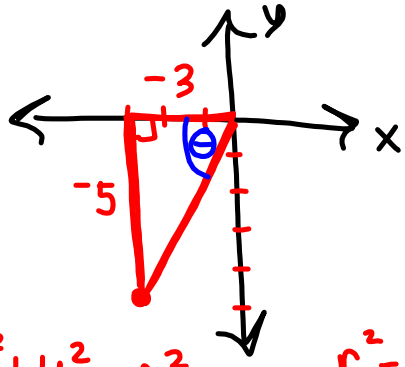
$$\tan\theta = \frac{y}{x}, \; x \neq 0 \qquad \cot\theta = \frac{x}{y}, \; y \neq 0.$$

The ratios in the second column are the reciprocals of the corresponding ratios in the first column.

EXAMPLE 1 Evaluating Trigonometric Functions

Let $P = (-3, -5)$ be a point on the terminal side of θ. Find each of the six trigonometric functions of θ.

x y



$x^2 + y^2 = r^2$

$(-3)^2 + (-5)^2 = r^2$

$9 + 25 = r^2$

$r^2 = 34$

$r = \sqrt{34}$

$\sin\theta = \frac{-5}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}} = \frac{-5\sqrt{34}}{34}$ $\csc\theta = -\frac{\sqrt{34}}{5}$

$\cos\theta = \frac{-3}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}} = \frac{-3\sqrt{34}}{34}$ $\sec\theta = -\frac{\sqrt{34}}{3}$

$\tan\theta = \frac{-5}{-3} = \frac{5}{3}$ $\cot\theta = \frac{3}{5}$

Check Point 1 Let $P = (1, -3)$ be a point on the terminal side of θ. Find each of the six trigonometric functions of θ.

$x \quad y$

$x^2+y^2=r^2$

$(1)^2+(-3)^2=r^2$

$1+9=r^2$

$10=r^2$

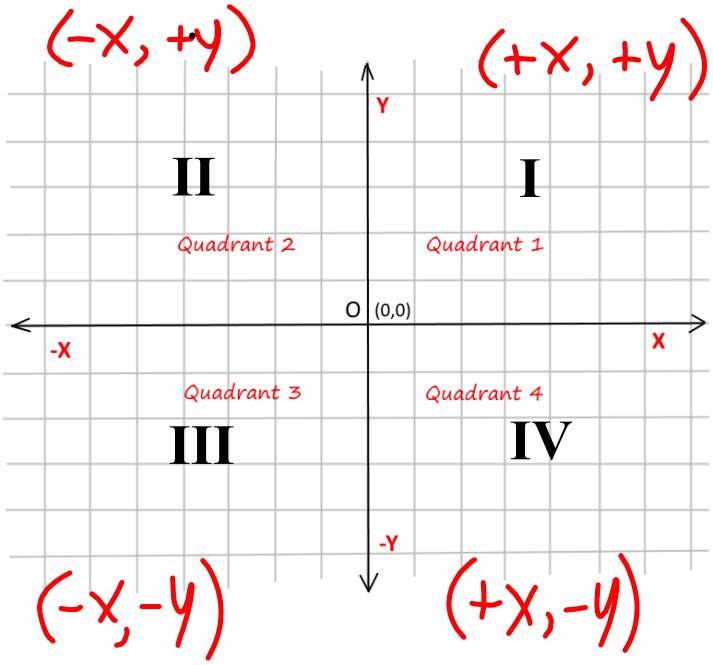
$\sqrt{10}=r$

$\sin\theta = \frac{-3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \boxed{\frac{-3\sqrt{10}}{10}}$ $\quad \csc\theta = \boxed{-\frac{\sqrt{10}}{3}}$

$\cos\theta = \frac{1}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \boxed{\frac{\sqrt{10}}{10}}$ $\quad \sec\theta = \boxed{\sqrt{10}}$

$\tan\theta = \frac{-3}{1} = \boxed{-3}$ $\quad \cot\theta = \boxed{-\frac{1}{3}}$

Let's Talk About Quadrants and Signs!



$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}, y \neq 0$
$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}, x \neq 0$
$\tan\theta = \frac{y}{x}, x \neq 0$	$\cot\theta = \frac{x}{y}, y \neq 0.$

* r is always positive!

EXAMPLE 3 Finding the Quadrant in Which an Angle Lies

If $\sin\theta > 0$ and $\cos\theta < 0$, name the quadrant in which angle θ terminates in.

y positive ↘ x negative

I or II ✓

If $\sin\theta < 0$ and $\cos\theta > 0$, name the quadrant in which angle θ terminates in.

y neg. x pos.

III or IV ✓

If $\tan\theta > 0$ and $\sin\theta < 0$, name the quadrant in which angle θ terminates in.

positive ↘ y neg.

$\frac{y}{x}$ or $\frac{-y}{-x}$

I or III ✓

If $\cot\theta < 0$ and $\cos\theta > 0$, name the quadrant in which angle θ terminates in.

negative ↘ x pos.

$\frac{-x}{y}$ or $\frac{x}{-y}$

II or IV ✓

EXAMPLE 3 Finding the Quadrant in Which an Angle Lies

If $\csc\theta < 0$ and $\cos\theta < 0$, name the quadrant in which angle θ terminates in.

$\frac{r}{-y}$ neg. ↘ x neg.

III ✓ or IV

If $\cot\theta < 0$ and $\csc\theta > 0$, name the quadrant in which angle θ terminates in.

$\frac{-x}{y}$ or $\frac{x}{-y}$ neg. $\frac{r}{+y}$ pos.

II ✓ or IV

NICE SUMMARY!

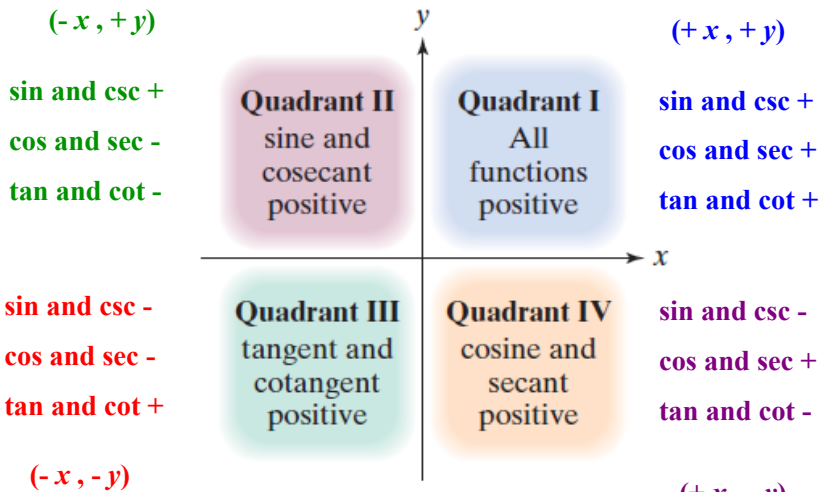


FIGURE 4.47 The signs of the trigonometric functions

HW 4.2 Day 1 --- p.548:

#'s: 1 - 7 odds, 17 - 22 all