

## 4.2 - Day 3 - Trig. Functions of Any Angle

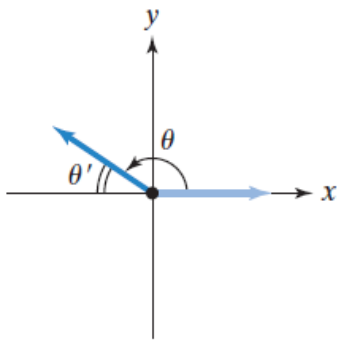
### Reference Angles

HW 4.2 Day 3 -

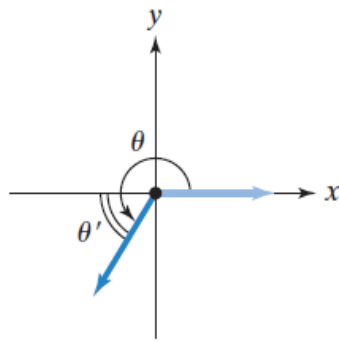
p.548: 35 - 47 odds, 51 - 59 odds

#### Definition of a Reference Angle

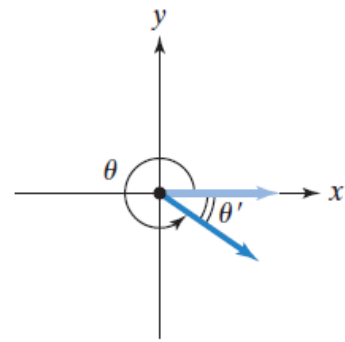
Let  $\theta$  be a nonacute angle in standard position that lies in a quadrant. Its **reference angle** is the positive acute angle  $\theta'$  formed by the terminal side of  $\theta$  and the  $x$ -axis.



If  $90^\circ < \theta < 180^\circ$ ,  
then  $\theta' = 180^\circ - \theta$ .



If  $180^\circ < \theta < 270^\circ$ ,  
then  $\theta' = \theta - 180^\circ$ .



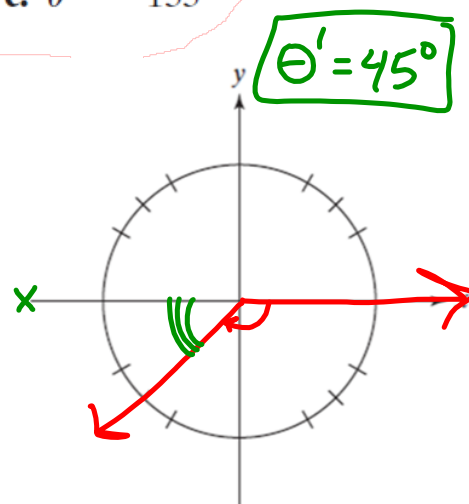
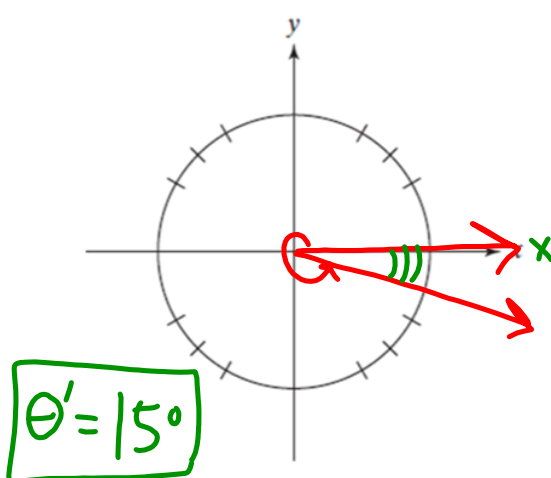
If  $270^\circ < \theta < 360^\circ$ ,  
then  $\theta' = 360^\circ - \theta$ .

### EXAMPLE 5 Finding Reference Angles

Find the reference angle,  $\theta'$ , for each of the following angles:

a.  $\theta = 345^\circ$

c.  $\theta = -135^\circ$

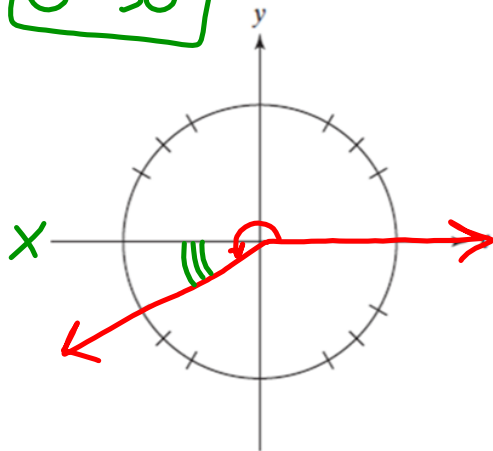


**Check Point 5** Find the reference angle,  $\theta'$ , for each of the following angles:

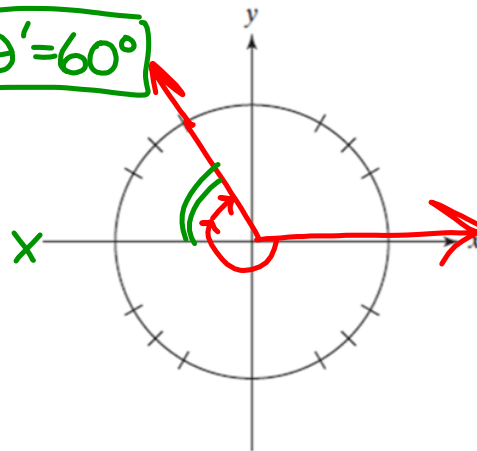
a.  $\theta = 210^\circ$

c.  $\theta = -240^\circ$

$\theta' = 30^\circ$



$\theta' = 60^\circ$



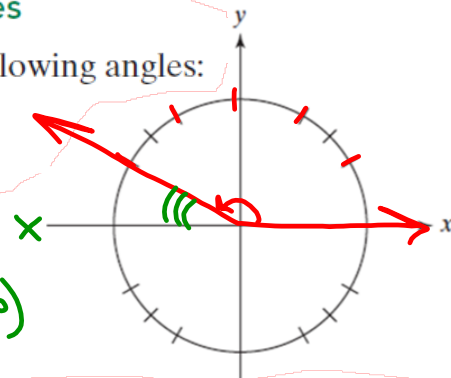
**EXAMPLE 5** Finding Reference Angles

Find the reference angle,  $\theta'$ , for each of the following angles:  
(In Radians)

b.  $\theta = \frac{5\pi}{6}$

$\theta' = \frac{\pi}{6}$

( $30^\circ$ )

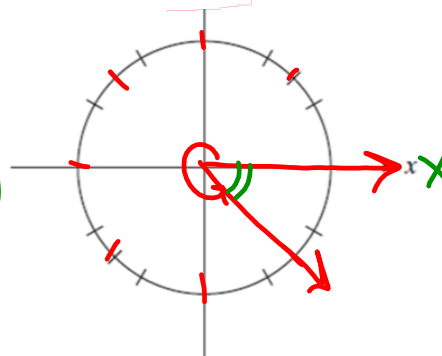


**Check Point 5** Find the reference angle,  $\theta'$ , for each of the following angles:  
(In Radians)

b.  $\theta = \frac{7\pi}{4}$

$\theta' = \frac{\pi}{4}$

( $45^\circ$ )



## Finding Reference Angles for Angles Greater Than $360^\circ$ ( $2\pi$ ) or Less Than $-360^\circ$ ( $-2\pi$ )

1. Find a positive angle  $\alpha$  less than  $360^\circ$  or  $2\pi$  that is coterminal with the given angle.
2. Draw  $\alpha$  in standard position.
3. Use the drawing to find the reference angle for the given angle. The positive acute angle formed by the terminal side of  $\alpha$  and the  $x$ -axis is the reference angle.

### EXAMPLE 6 Finding Reference Angles (Greater than $360^\circ$ or $2\pi$ )

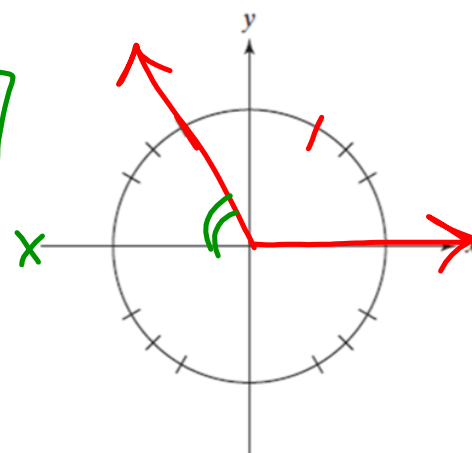
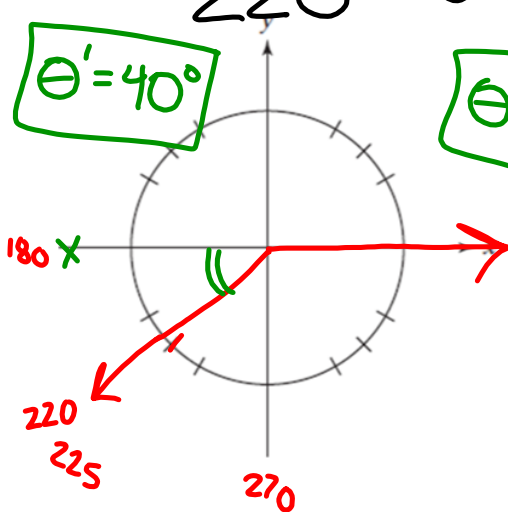
Find the reference angle for each of the following angles:

a.  $\theta = 580^\circ - 360^\circ$       b.  $\theta = \frac{8\pi}{3} - \frac{6\pi}{3}$

$220^\circ$        $\theta = \frac{2\pi}{3}$

$\theta' = 40^\circ$

$\theta' = \frac{\pi}{3}$

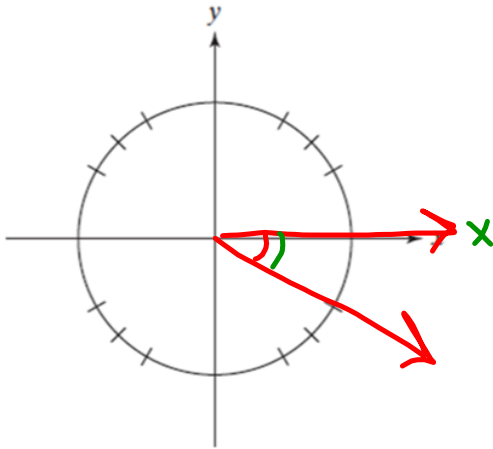


**EXAMPLE 6** Finding Reference Angles (Greater than  $360^\circ$  or  $2\pi$ )

Find the reference angle for each of the following angles:

$$\text{c. } \theta = -\frac{13\pi}{6} + \frac{12\pi}{6}$$

$$\theta' = -\frac{\pi}{6}$$



$$\theta' = \frac{\pi}{6}$$