4.2 - Day 4 - Trig. Functions of Any Angle

The Unit Circle

Recall: The **unit circle** is a circle of radius 1, with its center at the origin of a rectangular coordinate system. The equation of this unit circle is $x^2 + y^2 = 1$. **Figure 4.19** shows a unit circle with a central angle measuring t radians.
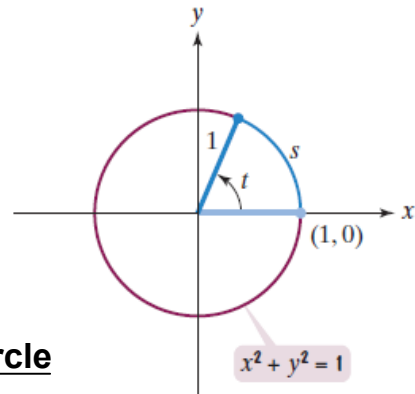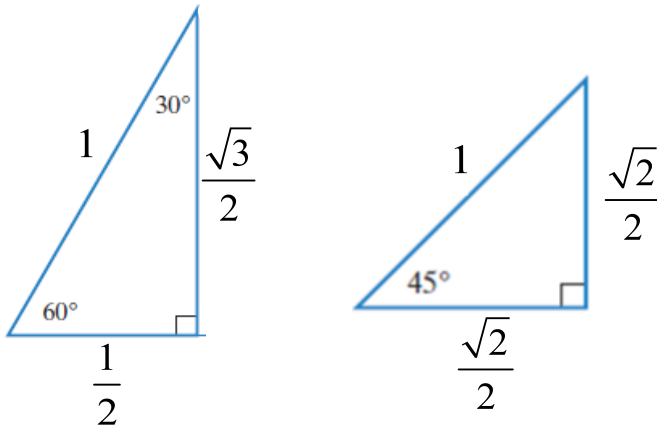
FIGURE 4.19 Unit circle with a central angle measuring t radians

Recall: Special Right Triangles in the Unit Circle

A Procedure for Using Reference Angles to Evaluate Trigonometric Functions

The value of a trigonometric function of any angle θ is found as follows:

1. Find the associated reference angle, θ', and the function value for θ'.
2. Use the quadrant in which θ lies to prefix the appropriate sign to the function value in step 1.

EXAMPLE 7 Using Reference Angles to Evaluate Trigonometric Functions

Use reference angles to find the exact value of each of the following trigonometric functions:

a. $\sin 135°$

QII
$(-x, +y)$

$\dfrac{y}{r} = \dfrac{\frac{\sqrt{2}}{2}}{1}$

$= \boxed{\dfrac{\sqrt{2}}{2}}$

b. $\cos 210°$

QIII $(-x, -y)$

$\dfrac{-x}{r} = \boxed{\dfrac{-\sqrt{3}}{2}}$

Use reference angles to find the exact value of

EXAMPLE

b. $\cos \dfrac{4\pi}{3}$

$\dfrac{\sqrt{2}}{2}$

45°

$\dfrac{\sqrt{2}}{2}$

1
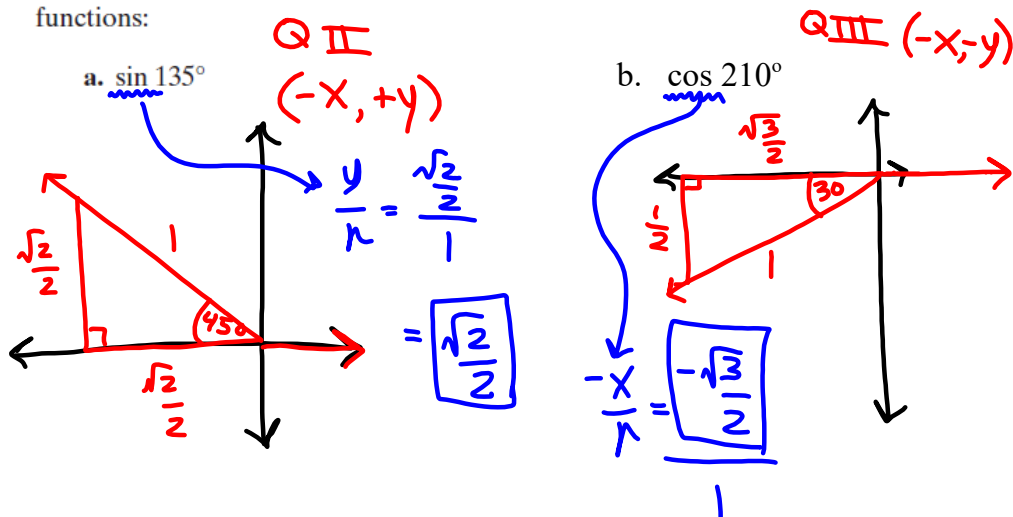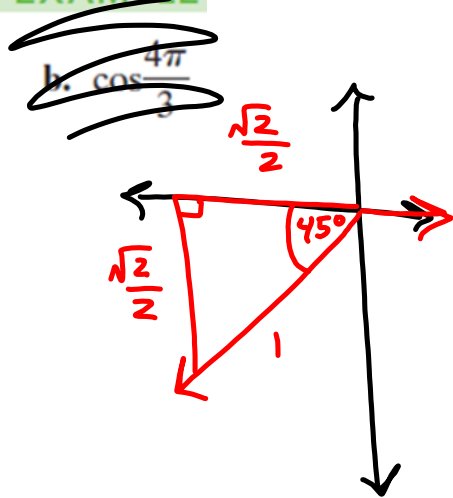
✅ Check Point

b. $\tan \dfrac{5\pi}{4}$

Q III

$(-x, -y)$

$\dfrac{-y}{-x} = \dfrac{\sqrt{2}}{2}$

$\dfrac{\dfrac{\sqrt{2}}{2}}{\dfrac{\sqrt{2}}{2}} = \boxed{1}$

Use reference angles to find the exact value of

EXAMPLE

c. $\cot\left(-\dfrac{\pi}{3}\right)$

Q IV $(+x, -y)$

$\dfrac{\sqrt{3}}{2}$

30°

1

$\dfrac{1}{2}$

✅ Check Point

c. $\sec\left(-\dfrac{\pi}{6}\right)$

$-30°$
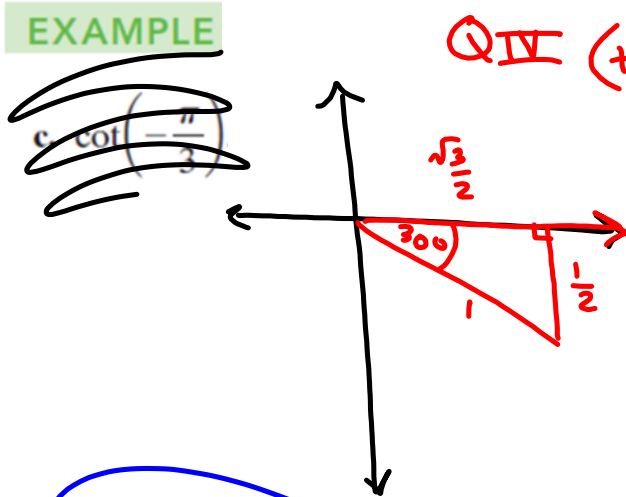
$\dfrac{r}{x} = \dfrac{1}{\dfrac{\sqrt{3}}{2}}$

$1 \cdot \dfrac{2}{\sqrt{3}} = \dfrac{2}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \boxed{\dfrac{2\sqrt{3}}{3}}$

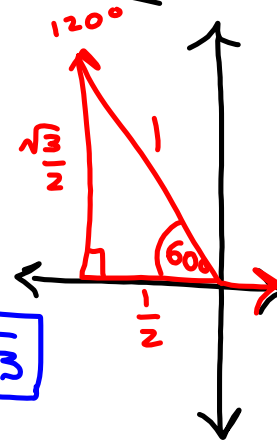Use reference angles to find the exact value of

EXAMPLE $Q\,II\,(-x,y)$ ~~Check Point~~

a. $\tan \dfrac{14\pi}{3} - \dfrac{6\pi}{3} - \dfrac{6\pi}{3}$ ~~b. sec 660°~~

$\tan \dfrac{2\pi}{3}$

$\dfrac{y}{-x} = \dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\dfrac{\sqrt{3}}{2} \cdot -\dfrac{2}{1} = -\dfrac{\sqrt{3}}{1} = \boxed{-\sqrt{3}}$

120°

$\dfrac{\sqrt{3}}{2}$ 1   60° $\dfrac{1}{2}$

Trigonometric Functions of Quadrantal Angles

Find the exact value of the following:

a.) $\cos \pi$ b.) $\tan \dfrac{\pi}{2}$

$(-1, 0)$ $(0, 1)$

$\dfrac{x}{r} = \dfrac{-1}{1} = \boxed{-1}$ $\dfrac{y}{x} = \dfrac{1}{0} = \boxed{\text{undef.}}$

$\dfrac{\pi}{2}$

$(0,1)$

1 s

π t

$(-1,0)$ $(1,0)$ x

$(0,-1)$

$x^2 + y^2 = 1$
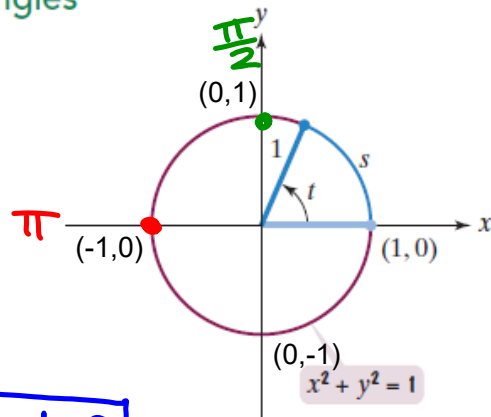
Trigonometric Functions of Quadrantal Angles

Find the exact value of the following:

c.) $\csc 2\pi$

$(1,0)$

$$\frac{r}{y} = \frac{1}{0} = \boxed{\text{undef.}}$$

d.) $\cot \dfrac{3\pi}{2}$

$(0,-1)$

$$\frac{x}{y} = \frac{0}{-1} = \boxed{0}$$

(0,1)

(-1,0)

(1,0) 2π

(0,-1)

$\dfrac{3\pi}{2}$

$x^2 + y^2 = 1$