4.3 - Nonlinear Systems of Equations - Day 1

Systems of Nonlinear Equations and Their Solutions

A **system of** two **nonlinear equations** in two variables, also called a **nonlinear system**, contains at least one equation that cannot be expressed in the form $Ax + By = C$. Here are two examples:

$$\begin{cases} x^2 = 2y + 10 \\ 3x - y = 9 \end{cases}$$

Not in the form $Ax + By = C$. The term x^2 is not linear.

$$\begin{cases} y = x^2 + 3 \\ x^2 + y^2 = 9. \end{cases}$$

Neither equation is in the form $Ax + By = C$. The terms x^2 and y^2 are not linear.

A **solution** of a nonlinear system in two variables is an ordered pair of real numbers that satisfies both equations in the system. The **solution set** of the system is the set of all such ordered pairs. As with linear systems in two variables, the solution of a nonlinear system (if there is one) corresponds to the intersection point(s) of the graphs of the equations in the system. Unlike linear systems, the graphs can be circles, parabolas, or anything other than two lines. We will solve nonlinear systems using the substitution method and the addition method.

HW 4.3 Day 1:
#'s: 2 - 18 evens, 50

EXAMPLE 1 Solving a Nonlinear System by the Substitution Method

Solve by the substitution method:

$$\begin{cases} x^2 = 2y + 10 & \text{(The graph is a parabola.)} \\ 3x - y = 9. & \text{(The graph is a line.)} \end{cases}$$

$3x - 9 = y$

$x^2 = 2(3x-9) + 10$

$x^2 = 6x - 18 + 10$

$x^2 = 6x - 8$

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$x = 4 \quad x = 2$

$\boxed{(4, 3)}$

$\boxed{(2, -3)}$

January 16, 2020

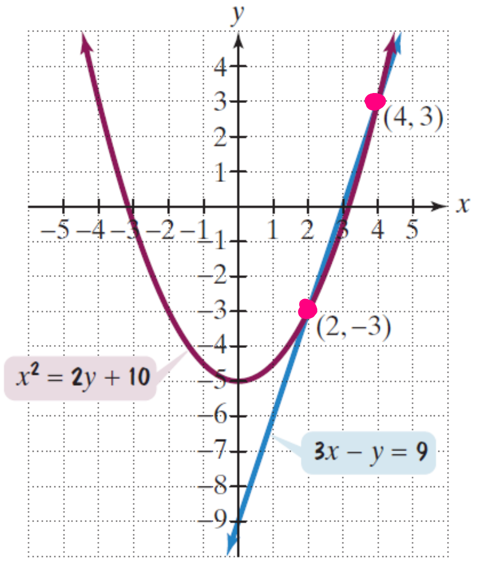

FIGURE 7.10 Points of intersection illustrate the nonlinear system's solutions.

Check Point 1 Solve by the substitution method:

$$\begin{cases} x^2 = y - 1 \\ 4x - y = -1. \end{cases}$$

$4x + 1 = y$

$x^2 = 4x + 1 - 1$

$x^2 = 4x$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0, \quad x = 4$

$\boxed{(0, 1)}$

$\boxed{(4, 17)}$

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EXAMPLE 2 Solving a Nonlinear System by the Substitution Method

Solve by the substitution method:

$x = y+3$

$$\begin{cases} x - y = 3 & \text{(The graph is a line.)} \\ (x-2)^2 + (y+3)^2 = 4. & \text{(The graph is a circle.)} \end{cases}$$

$(y+3-2)^2 + (y+3)^2 = 4$

$(y+1)^2 + (y+3)^2 = 4$

$y^2+2y+1 + y^2+6y+9 = 4$

$2y^2+8y+10=4$

$\frac{2y^2}{2}+\frac{8y}{2}+\frac{6}{2}=\frac{0}{2}$

$y^2+4y+3=0$

$(y+3)(y+1)=0$

$y=-3, \; y=-1$

$x = y+3$

$\boxed{(2,-1)}$

$\boxed{(0,-3)}$

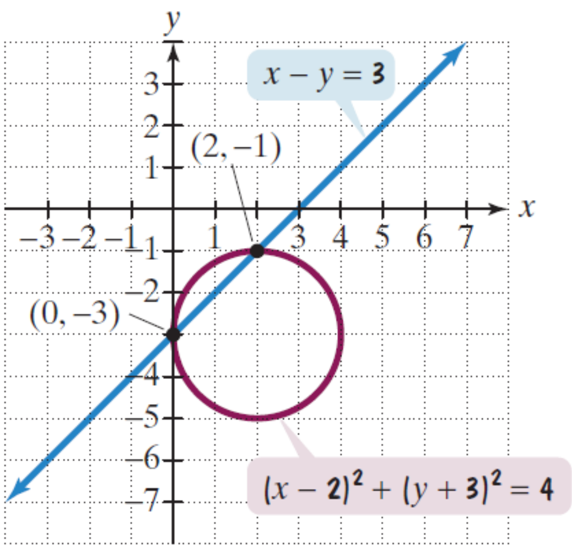

FIGURE 7.11 Points of intersection illustrate the nonlinear system's solutions.