

- 4.5 - Trig. Identities - Day 1

A **trigonometric identity** in one variable is a trigonometric equation that is true for all values of the variable for which all expressions in the equation are defined.

Essential Understanding The interrelationships among the six basic trigonometric functions make it possible to write trigonometric expressions in various equivalent forms, some of which can be significantly easier to work with than others in mathematical applications.

Some trigonometric identities are definitions or follow immediately from definitions.

take note

Key Concept Basic Identities

Reciprocal Identities $\csc \theta = \frac{1}{\sin \theta}$

$\sec \theta = \frac{1}{\cos \theta}$

Also: $\frac{1}{\csc \theta} = \sin \theta$

Also: $\frac{1}{\sec \theta} = \cos \theta$

Tangent Identity $\tan \theta = \frac{\sin \theta}{\cos \theta}$

Cotangent Identity $\cot \theta = \frac{\cos \theta}{\sin \theta}$

Also: $\frac{1}{\cot \theta} = \tan \theta$

Also: $\frac{1}{\tan \theta} = \cot \theta$

EXAMPLE 1 Changing to Sines and Cosines to Verify an Identity

Verify the identity: $\sec x \cot x = \csc x$.

1. $\frac{1}{\cos x} \cdot \frac{\cos x}{\sin x}$

2. $\frac{\cancel{\cos x}}{\cancel{\cos x} \cdot \sin x}$

3. $\frac{1}{\sin x}$

4. $\csc x$ ✓

✓ Check Point 1 Verify the identity: $\underline{\csc x} \underline{\tan x} = \sec x$.

1. $\frac{1}{\sin x} \cdot \frac{\sin x}{\cos x}$
2. $\frac{\cancel{\sin x}}{\cancel{\sin x} \cdot \cos x}$
3. $\frac{1}{\cos x}$
4. $\sec x$ ✓

Additional Examples:

Verifying an Identity Using Basic Identities
(note: some problems use theta for the variable)

$$\sin \theta \underline{\sec \theta} = \tan \theta$$

$$\cos x \underline{\csc x} \underline{\tan x} = 1$$

1. $\sin \theta \cdot \frac{1}{\cos \theta}$

1. $\cos x \cdot \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x}$

2. $\frac{\sin \theta}{\cos \theta}$

2. $\frac{\cancel{\cos x} \cdot \cancel{\sin x}}{\cancel{\sin x} \cdot \cancel{\cos x}}$

3. $\tan \theta$ ✓

3. 1 ✓

Additional Example:**Verifying an Identity Using Basic Identities**

$$\frac{\csc \theta}{\sec \theta} = \cot \theta$$

1.
$$\frac{1}{\sin \theta}$$

$$\frac{1}{\cos \theta}$$

2.
$$\frac{1}{\sin \theta} \cdot \frac{\cos \theta}{1}$$

"multiply by recip."

3.
$$\frac{\cos \theta}{\sin \theta}$$

4. $\cot \theta$ ✓