March 05, 2020

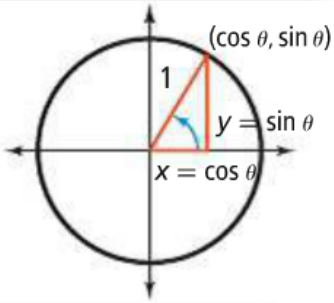
- 4.5 - Trig. Identities - Day 2

take note

Key Concept Pythagorean Identities

$\cos^2\theta + \sin^2\theta = 1$ $\quad 1 + \tan^2\theta = \sec^2\theta$ $\quad 1 + \cot^2\theta = \csc^2\theta$

You can use the unit circle and the Pythagorean Theorem to verify another identity. The circle with its center at the origin with a radius of 1 is called the unit circle, and has an equation $x^2 + y^2 = 1$.



Every angle θ determines a unique point on the unit circle with x- and y-coordinates $(x, y) = (\cos\theta, \sin\theta)$.

Therefore, for every angle θ,

$(\cos\theta)^2 + (\sin\theta)^2 = 1$ or $\cos^2\theta + \sin^2\theta = 1$.

This form allows you to write the identity without using parentheses.

EXAMPLE 2 Changing to Sines and Cosines to Verify an Identity

Verify the identity: $\sin x \tan x + \cos x = \sec x$.

1. $\sin x \cdot \frac{\sin x}{\cos x} + \cos x$

2. $\frac{\sin^2 x}{\cos x} + \frac{\cos x}{1} \cdot \frac{\cos x}{\cos x}$

3. $\frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x}$

4. $\frac{\sin^2 x + \cos^2 x}{\cos x}$

5. $\frac{1}{\cos x}$

6. $\sec x$ ✓

Check Point 2 Verify the identity: $\cos x \cot x + \sin x = \csc x.$

1. $\cos x \cdot \frac{\cos x}{\sin x} + \sin x$
2. $\frac{\cos^2 x}{\sin x} + \frac{\sin x}{1} \cdot \frac{\sin x}{\sin x}$
3. $\frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x}$
4. $\frac{\cos^2 x + \sin^2 x}{\sin x}$
5. $\frac{1}{\sin x}$
6. $\csc x$ ✓

EXAMPLE 3 Using Factoring to Verify an Identity

Verify the identity: $\cos x - \cos x \sin^2 x = \cos^3 x.$

GCF out a 'cos x'

1. $\cos x(1 - \sin^2 x)$
2. $\cos x(\cos^2 x)$
3. $\cos^3 x$ ✓

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EXAMPLE 4 Using Two Techniques to Verify an Identity

Verify the identity: $\dfrac{1 + \sin\theta}{\cos\theta} = \sec\theta + \tan\theta.$

$\dfrac{a+b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$

1. $\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

2. $\sec\theta + \tan\theta$ ✓

$\dfrac{5+3}{2} = \dfrac{5}{2} + \dfrac{3}{2} = \dfrac{8}{2}$