4.7 - Trigonometric Equations - Day 3
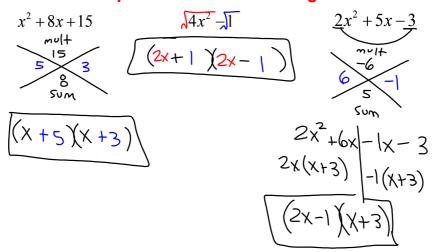
Trigonometric Equations Quadratic in Form

Some trigonometric equations are in the form of a quadratic equation $au^2 + bu + c = 0$, where u is a trigonometric function and $a \neq 0$. Here are two examples of trigonometric equations that are quadratic in form:

$$2\cos^2 x + \cos x - 1 = 0 \qquad 2\sin^2 x - 3\sin x + 1 = 0.$$

The form of this equation is $2u^2 + u - 1 = 0$ with $u = \cos x$.

The form of this equation is $2u^2 - 3u + 1 = 0$ with $u = \sin x$.

~~HW Day3 4.7:~~
~~p. 674. 39 - 51 odds~~

To solve this kind of equation, try using factoring. If the trigonometric expression does not factor, use another method, such as the quadratic formula or the square root property.

Lets practice a little factoring first!

$x^2 + 8x + 15$

mult 15

5   3

8 sum

$(x+5)(x+3)$

$\sqrt{4x^2} - \sqrt{1}$

$(2x+1)(2x-1)$

$2x^2 + 5x - 3$

mult -6

6   -1

5 sum

$2x^2 + 6x - 1x - 3$

$2x(x+3) \quad -1(x+3)$

$(2x-1)(x+3)$

EXAMPLE 4   Solving a Trigonometric Equation Quadratic in Form

Solve the equation: $2\cos^2 x + \cos x - 1 = 0, \quad 0 \le x < 2\pi$.

$$(2\cos x - 1)(\cos x + 1) = 0$$

$2\cos x - 1 = 0$ $\cos x + 1 = 0$

$2\cos x = 1$ $\cos x = -1$

$\cos x = \dfrac{1}{2}$

$x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$ $x = \pi$

1

✓ **Check Point 4** Solve the equation: $2 \sin^2 x - 3 \sin x + 1 = 0, \quad 0 \le x < 2\pi.$

$$(2 \sin x - 1)(\sin x - 1) = 0$$

$2 \sin x - 1 = 0 \qquad\qquad \sin x - 1 = 0$

$2 \sin x = 1 \qquad\qquad \sin x = 1$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{2}$$

unit
circles: $x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

✓ **Check Point 5** Solve the equation: $4 \cos^2 x - 3 = 0, \quad 0 \le x < 2\pi.$

$4 \cos^2 x - 3 = 0$

$4 \cos^2 x = 3$

$$\sqrt{\cos^2 x} = \sqrt{\frac{3}{4}}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

"where are the x-values
$\pm \dfrac{\sqrt{3}}{2}$?"

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$