Unit #8 - 4.8 - Law of Sines - Day 1

The Law of Sines and Its Derivation

An **oblique triangle** is a triangle that does not contain a right angle. **Figure 6.1** shows that an oblique triangle has either three acute angles or two acute angles and one obtuse angle. Notice that the angles are labeled A, B, and C. The sides opposite each angle are labeled as a, b, and c, respectively.

HW 4.8 Day1:
p.690: 2 - 8 evens, 45, 46, 47

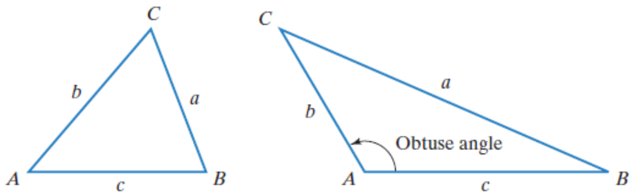


FIGURE 6.1 Oblique triangles

The relationships among the sides and angles of right triangles defined by the trigonometric functions are not valid for oblique triangles. Thus, we must observe and develop new relationships in order to work with oblique triangles.

Many relationships exist among the sides and angles in oblique triangles. One such relationship is called the **Law of Sines**.

The Law of Sines

If A, B, and C are the measures of the angles of a triangle, and a, b, and c are the lengths of the sides opposite these angles, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

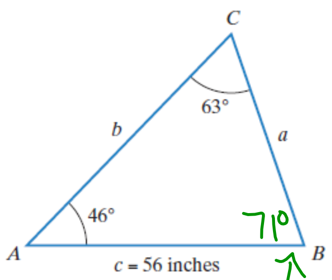
The ratio of the length of the side of any triangle to the sine of the angle opposite that side is the same for all three sides of the triangle.

Solving Oblique Triangles

Solving an oblique triangle means finding the lengths of its sides and the measurements of its angles. The Law of Sines can be used to solve a triangle in which one side and two angles are known. The three known measurements can be abbreviated using SAA (a side and two angles are known) or ASA (two angles and the side between them are known).

EXAMPLE 1 Solving an SAA Triangle Using the Law of Sines

Solve the triangle shown in **Figure 6.3** with $A = 46°$, $C = 63°$, and $c = 56$ inches. Round lengths of sides to the nearest tenth.



$$\frac{56}{\sin(63)} = \frac{a}{\sin(46)}$$

$$\frac{a \cdot \sin(63)}{\sin(63)} = \frac{40.2830}{\sin(63)}$$

$$\boxed{a = 45.2 \text{ in.}}$$

$180 - (46+63)$

$$\boxed{\angle B = 71°}$$

$$\frac{56}{\sin(63)} = \frac{b}{\sin(71)}$$

$$\boxed{b = 59.4 \text{ in.}}$$

Check Point 1 Solve the triangle shown in **Figure 6.4** with $A = 64°$, $C = 82°$, and $c = 14$ centimeters. Round as in Example 1.

$\angle B = 180 - (82 + 64)$

$\boxed{\angle B = 34°}$

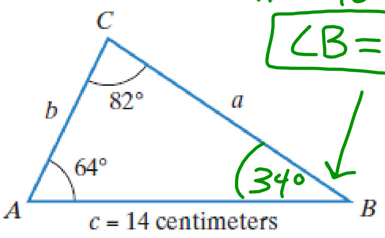


FIGURE 6.4

$$\frac{14}{\sin(82)} = \frac{a}{\sin(64)}$$

$\boxed{a = 12.7 \text{ cm}}$

$$\frac{14}{\sin(82)} = \frac{b}{\sin(34)}$$

$\boxed{b = 7.9 \text{ cm}}$

EXAMPLE 2 Solving an ASA Triangle Using the Law of Sines

Solve triangle ABC if $A = 50°$, $C = 33.5°$, and $b = 76$. Round measures to the nearest tenth.

$$\frac{76}{\sin(96.5)} = \frac{a}{\sin(50)}$$

$\boxed{a = 58.6}$

$$\frac{76}{\sin(96.5)} = \frac{c}{\sin(33.5)}$$

$\boxed{c = 42.2}$

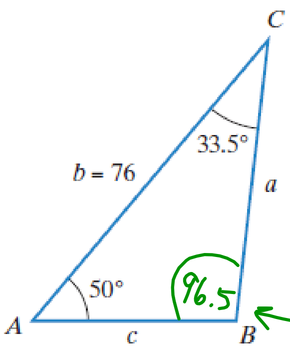


FIGURE 6.5 Solving an ASA triangle

$\angle B = 180 - (50 + 33.5)$

$\boxed{\angle B = 96.5°}$

Check Point 2 Solve triangle ABC if $A = 40°$, $C = 22.5°$, and $b = 12$. Round as in Example 2.