# 4.8 - Law of Sines - Day 2

HW 4.8 Day2:
p.690: 17 - 31 odds

## The Ambiguous Case (SSA)

If we are given two sides and an angle opposite one of them (SSA), does this determine a unique triangle? Can we solve this case using the Law of Sines? Such a case is called the **ambiguous case** because the given information may result in one triangle, two triangles, or no triangle at all. For example, in **Figure 6.6**, we are given $a$, $b$, and $A$. Because $a$ is shorter than $h$, it is not long enough to form a triangle. The number of possible triangles, if any, that can be formed in the SSA case depends on $h$, the length of the altitude, where $h = b \sin A$.

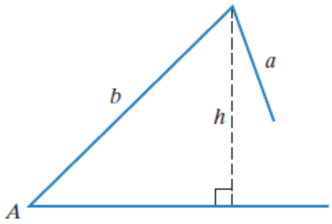
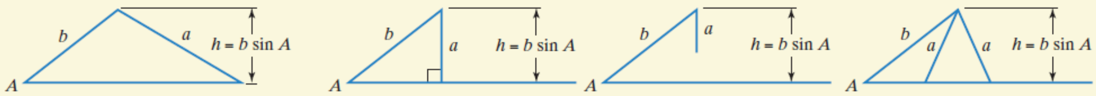



FIGURE 6.6 Given SSA, no triangle may result.

### The Ambiguous Case (SSA)

Consider a triangle in which $a$, $b$, and $A$ are given. This information may result in

**One Triangle** | **One Right Triangle** | **No Triangle** | **Two Triangles**




- **One Triangle:** $a$ is greater than $h$ and $a$ is greater than $b$. One triangle is formed.
- **One Right Triangle:** $a = h$ and is just the right length to form a right triangle.
- **No Triangle:** $a$ is less than $h$ and is not long enough to form a triangle.
- **Two Triangles:** $a$ is greater than $h$ and $a$ is less than $b$. Two distinct triangles are formed.

## EXAMPLE 3 Solving an SSA Triangle Using the Law of Sines (One Solution)

Solve triangle $ABC$ if $A = 43°$, $a = 81$, and $b = 62$. Round lengths of sides to the nearest tenth and angle measures to the nearest degree.

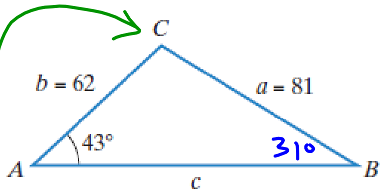



FIGURE 6.7 Solving an SSA triangle; the ambiguous case

I.

$$\frac{81}{\sin(43)} = \frac{62}{\sin B}$$

$$\frac{81 \cdot \sin B}{81} = \frac{42.284}{81}$$

$$\sin B = .5220$$

$$\sin^{-1}(.5220) = B$$

$$\boxed{\angle B = 31°}$$

II.

$$\angle C = 180 - (31 + 43)$$

$$\boxed{\angle C = 106°}$$

III.

$$\frac{81}{\sin(43)} = \frac{c}{\sin(106)}$$

$$\boxed{c = 114.2}$$

**EXAMPLE 4** Solving an SSA Triangle Using the Law of Sines (No Solution)

Solve triangle $ABC$ if $A = 75°$, $a = 51$, and $b = 71$.

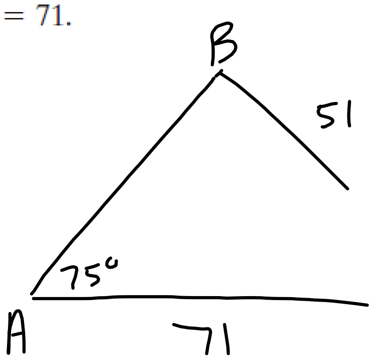
$$\frac{51}{\sin(75)} = \frac{71}{\sin B}$$

$$51 \cdot \sin B = 68.5807$$

$$\sin B = 1.3447$$

$$\sin^{-1}(1.3447) = B$$

$$\angle B = \text{undef.}$$




$\boxed{\text{No triangle}}$

**EXAMPLE 5** Solving an SSA Triangle Using the Law of Sines (Two Solutions)

Solve triangle $ABC$ if $A = 40°$, $a = 54$, and $b = 62$. Round lengths of sides to the nearest tenth and angle measures to the nearest degree.

$$\frac{54}{\sin(40)} = \frac{62}{\sin B}$$

$$\sin B = .7380$$

• CASE 2: $\boxed{\angle B = 48°}$

$$\angle C = 180 - (48 + 40)$$

$$\boxed{\angle C = 92°}$$

$$\frac{54}{\sin(40)} = \frac{c}{\sin(92)} \quad \boxed{c = 84}$$

OR

• CASE 2:

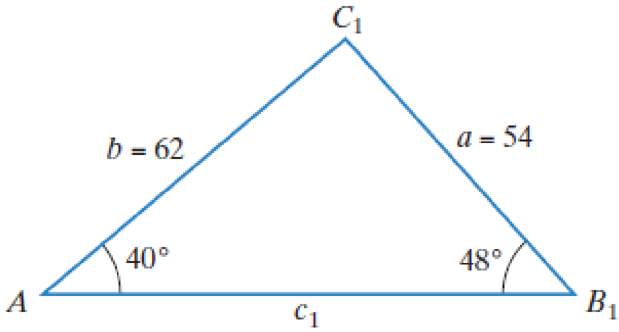
$$\angle B = 180 - 48$$

$$\boxed{\angle B = 132°}$$

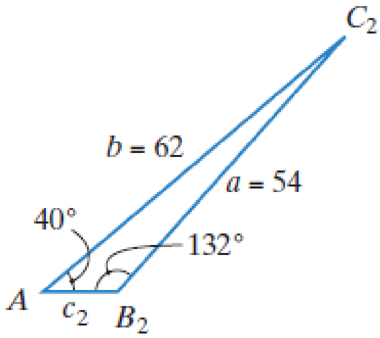
$$\angle C = 180 - (132 + 40)$$

$$\boxed{\angle C = 8°}$$

$$\frac{54}{\sin(40)} = \frac{c}{\sin(8)} \quad \boxed{c = 11.7}$$




**(b)** In one possible triangle, $B_1 = 48°$.




**(c)** In the second possible triangle, $B_2 = 132°$.