4.8 - Law of Cosines - Day 3

The Law of Cosines

We now look at another relationship that exists among the sides and angles in an oblique triangle. The **Law of Cosines** is used to solve triangles in which two sides and the included angle (SAS) are known, or those in which three sides (SSS) are known.

The Law of Cosines

If A, B, and C are the measures of the angles of a triangle, and a, b, and c are the lengths of the sides opposite these angles, then

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C.$$

The square of a side of a triangle equals the sum of the squares of the other two sides minus twice their product times the cosine of their included angle.

Solving Oblique Triangles

If you are given two sides and an included angle (SAS) of an oblique triangle, none of the three ratios in the Law of Sines is known. This means that we do not begin solving the triangle using the Law of Sines. Instead, we apply the Law of Cosines and the following procedure:

Solving an SAS Triangle

1. Use the Law of Cosines to find the side opposite the given angle.
2. Use the Law of Sines to find the angle opposite the shorter of the two given sides. This angle is always acute.
3. Find the third angle by subtracting the measure of the given angle and the angle found in step 2 from 180°.

EXAMPLE 1 Solving an SAS Triangle

Solve the triangle in **Figure 6.15** with $A = 60°$, $b = 20$, and $c = 30$. Round lengths of sides to the nearest tenth and angle measures to the nearest degree.

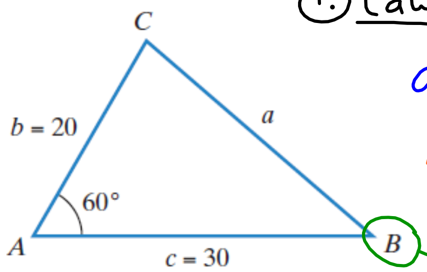


FIGURE 6.15 Solving an SAS triangle

1. Law of Cosines:

$a^2 = 20^2 + 30^2 - 2\cdot 20\cdot 30\cdot \cos(60)$

$\sqrt{a^2} = \sqrt{700}$ → $\boxed{a = 26.5}$

2. Law of Sines:

$\frac{20}{\sin B} = \frac{26.5}{\sin(60)}$

$\sin B = .6536.....$

$\boxed{\angle B = 41°}$

3. Last angle:

$180 - (41 + 60)$

$\boxed{\angle C = 79°}$

Check Point 1 Solve the triangle shown in **Figure 6.16** with $A = 120°$, $b = 7$, and $c = 8$. Round as in Example 1.

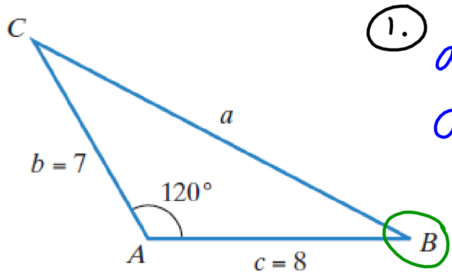


FIGURE 6.16

1. $a^2 = 7^2 + 8^2 - 2\cdot 7\cdot 8\cdot \cos(120)$

$a^2 = 169$, $\boxed{a = 13}$

2. $\frac{7}{\sin B} = \frac{13}{\sin(120)}$

$\boxed{\angle B = 28°}$

3. $180 - (120 + 28)$

$\boxed{\angle C = 32°}$

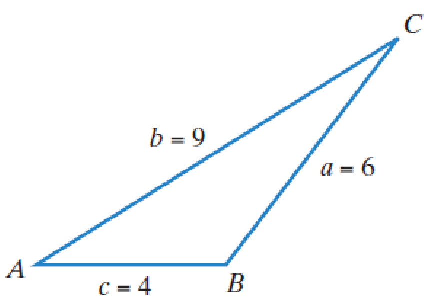
If you are given three sides of a triangle (SSS), solving the triangle involves finding the three angles. We use the following procedure:

Solving an SSS Triangle

1. Use the Law of Cosines to find the angle opposite the longest side.
2. Use the Law of Sines to find either of the two remaining acute angles.
3. Find the third angle by subtracting the measures of the angles found in steps 1 and 2 from 180°.

EXAMPLE 2 Solving an SSS Triangle

Solve triangle ABC if $a = 6$, $b = 9$, and $c = 4$. Round angle measures to the nearest degree.



① Find ∠B w/ Law of Cosines:

$9^2 = 6^2 + 4^2 - 2 \cdot 6 \cdot 4 \cdot \cos B$

$81 = 52 - 48 \cdot \cos B$

$-52 \quad -52$

$29 = -48 \cdot \cos B$

$-48 \quad -48$

$-.60416.... = \cos B$

$\boxed{\angle B = 127°}$

② Law of Sines:

$\frac{6}{\sin A} = \frac{9}{\sin(127)}$

$\boxed{\angle A = 32°}$

③ Last angle:

$180 - (32 + 127)$ $\boxed{\angle C = 21°}$

Check Point 2 Solve triangle ABC if $a = 8$, $b = 10$, and $c = 5$. Round angle measures to the nearest degree.

$\boxed{1.}$ $10^2 = 8^2 + 5^2 - 2\cdot8\cdot5\cdot\cos B$

$100 = 89 - 80\cdot\cos B$

$-89 \quad -89$

$\frac{11}{-80} = \frac{-80\cdot\cos B}{-80}$

$-.1375 = \cos B$

$\boxed{\angle B = 98^\circ}$

$\boxed{2.}$ $\angle A = ?$

$\frac{8}{\sin A} = \frac{10}{\sin(98)}$

$\boxed{\angle A = 52^\circ}$

$\boxed{3.}$

$180 - (98 + 52)$

$\boxed{\angle C = 30^\circ}$

HW - Law of Cosines - Day 3

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