# UNIT 5 --- 5.1 - Day 1 - Polynomial Functions

## Definition of a Polynomial Function

Let $n$ be a nonnegative integer and let $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ be real numbers, with $a_n \neq 0$. The function defined by

$$f(x) = \boxed{a_n x^n} + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$$

is called a **polynomial function of degree *n***. The number $a_n$, the coefficient of the variable to the highest power, is called the **leading coefficient**.

**Polynomial Functions**

$$f(x) = -3x^5 + \sqrt{2}x^2 + 5$$

Polynomial function of degree 5

$$\begin{aligned} g(x) &= -3x^4(x-2)(x+3) \\ &= -3x^4(x^2+x-6) \\ &= -3x^6 - 3x^5 + 18x^4 \end{aligned}$$

Polynomial function of degree 6

**Not Polynomial Functions**

$$\begin{aligned} F(x) &= -3\sqrt{x} + \sqrt{2}x^2 + 5 \\ &= -3x^{\frac{1}{2}} + \sqrt{2}x^2 + 5 \end{aligned}$$

The exponent on the variable is not an integer.

$$\begin{aligned} G(x) &= -\frac{3}{x^2} + \sqrt{2}x^2 + 5 \\ &= -3x^{-2} + \sqrt{2}x^2 + 5 \end{aligned}$$

The exponent on the variable is not a nonnegative integer.

A constant function $f(x) = c$, where $c \neq 0$, is a polynomial function of degree 0. A linear function $f(x) = mx + b$, where $m \neq 0$, is a polynomial function of degree 1. A quadratic function $f(x) = ax^2 + bx + c$, where $a \neq 0$, is a polynomial function of degree 2. In this section, we focus on polynomial functions of degree 3 or higher.

## Classifying Polynomials~

| Degree | Name Using Degree | Polynomial Example | Number of Terms | Name Using Number of Terms |
|---|---|---|---|---|
| 0 | constant | $5$ | 1 | monomial |
| 1 | linear | $x + 4$ | 2 | binomial |
| 2 | quadratic | $4x^2$ | 1 | monomial |
| 3 | cubic | $4x^3 - 2x^2 + x$ | 3 | trinomial |
| 4 | quartic | $2x^4 + 5x^2$ | 2 | binomial |
| 5 | quintic | $-x^5 + 4x^2 + 2x + 1$ | 4 | polynomial of 4 terms |

Write each polynomial in standard form. What is the classification of each by degree? by number of terms?

**a.** $3x^3 - x + 5x^4$

$5x^4 + 3x^3 - x$

$\boxed{\text{Quartic Trinomial}}$

**b.** $3 - 4x^5 + 2x^2 + 10$

$-4x^5 + 2x^2 + 13$

$\boxed{\text{Quintic Trinomial}}$

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**Classify. Get in standard form first.**

$3x^2 - 2x^2 + 7x^2$

$\boxed{8x^2}$

Quadratic Monomial

$x(x - 2) + x^2(x + 3) + 2x$

$x^2 - 2x + x^3 + 3x^2 + 2x$

$\boxed{x^3 + 4x^2}$

Cubic Binomial

## Smooth, Continuous Graphs

Polynomial functions of degree 2 or higher have graphs that are *smooth* and *continuous*. By **smooth**, we mean that the graphs contain only rounded curves with no sharp corners. By **continuous**, we mean that the graphs have no breaks and can be drawn without lifting your pencil from the rectangular coordinate system. These ideas are illustrated in **Figure 2.14**.

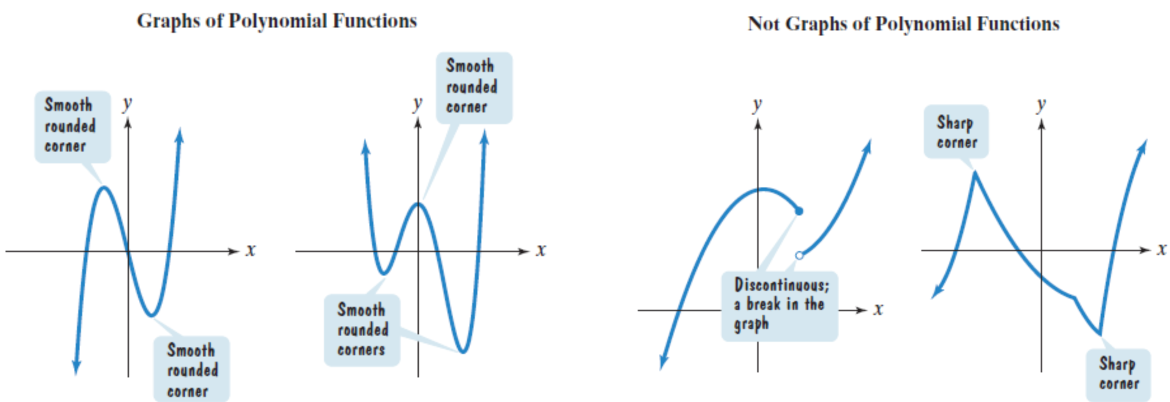



FIGURE 2.14 Recognizing graphs of polynomial functions

# End Behavior of Polynomial Functions

## The Leading Coefficient Test

As $x$ increases or decreases without bound, the graph of the polynomial function

$$f(x) = \boxed{a_n x^n} + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0 \quad (a_n \neq 0)$$

eventually rises or falls. In particular,

**1.** For $n$ odd:

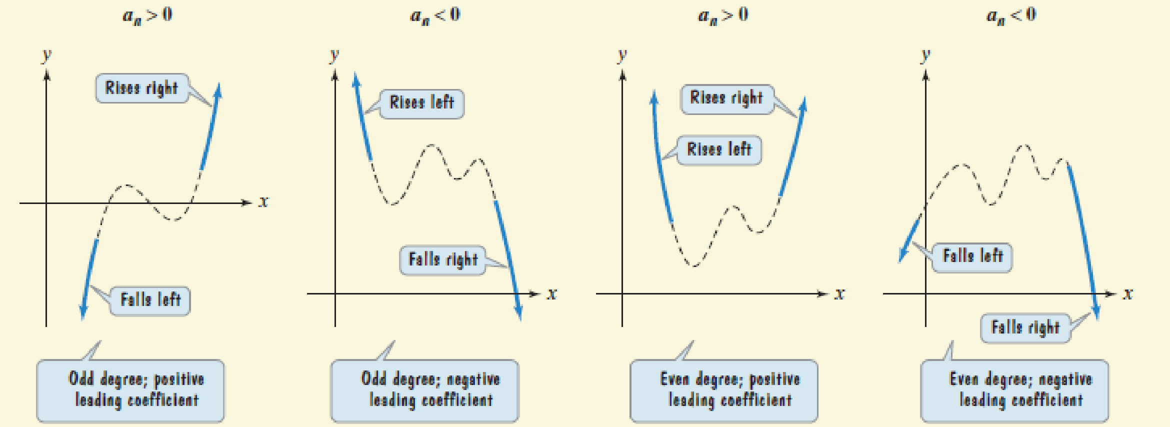
If the leading coefficient is positive, the graph falls to the left and rises to the right. (↙, ↗)

If the leading coefficient is negative, the graph rises to the left and falls to the right. (↖, ↘)

**2.** For $n$ even:

If the leading coefficient is positive, the graph rises to the left and rises to the right. (↖, ↗)

If the leading coefficient is negative, the graph falls to the left and falls to the right. (↙, ↘)




## EXAMPLE 1 Using the Leading Coefficient Test

Use the Leading Coefficient Test to determine the end behavior of the graph of

$$f(x) = x^3 + 3x^2 - x - 3.$$

a: +

down/up

**Check Point 1** Use the Leading Coefficient Test to determine the end behavior of the graph of $f(x) = x^4 - 4x^2$.

a: +

up/up

## EXAMPLE 2 Using the Leading Coefficient Test

Use the Leading Coefficient Test to determine the end behavior of the graph of

$$f(x) = -4x^3(x-1)^2(x+5).$$

degree

3+2+1 = 6

a: − (neg.)

down/down

**Check Point 2** Use the Leading Coefficient Test to determine the end behavior of the graph of $f(x) = 2x^3(x-1)(x+5)$.

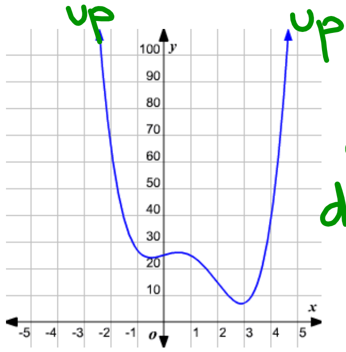
a: +

degree 5

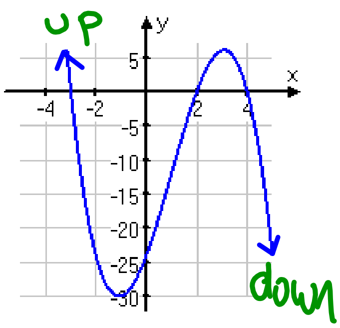
down/up

## Determine End Behavior~

Given a graph of a polynomial, determine if the degree is even or odd, and what the sign of the leading coefficient (a-value) is.

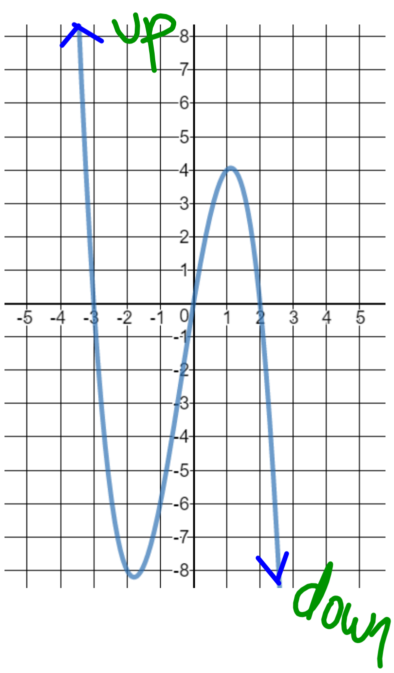



a: $\boxed{\text{positive}}$
degree: $\boxed{\text{even}}$




a: $\boxed{\text{neg.}}$
degree: $\boxed{\text{odd}}$

## SAT-Style-Question:




Which of these equations could represent the graph?

~~a.~~ $y = x^2 + x - 6$

→ ~~b.~~ $y = -x^2 - x + 6$ even

~~c.~~ $y = x^3 + x^2 - 6x$

→ d. $y = -x^3 - x^2 + 6x$

odd